PHYSICS HOMEWORK #111 SPECIAL RELATIVITY

LENGTH CONTRACTION

$$L = L_o\sqrt{1-(v^2/c^2)}$$

WHERE L_o = Length measured by an observer at rest relative to the length being measured
AND L = Length measured by an observer moving with a velocity V relative to the length being measured
AND WHERE c = 3.0 x 10^8 m/s [the speed of light in a vacuum]

1. A rocket at rest on Earth is measured to have a length of 45.0 meters and a diameter of 5.60 meters. The rocket is then launched toward Alpha Centauri, which is 4.3 light years from the Earth, at 2.2 x 10^8 m/s.
   a. What will be the length of this rocket as measured by an observer on the Earth?
   b. What will be the length of the rocket as measured by an astronaut on board the rocket?
   c. What will be the diameter of the rocket as measured by an observer on the Earth?
   d. What will be the distance to Alpha Centauri as measured by the astronaut on board the rocket?

2. The distance to the star Epsilon Indi, as measured from the Earth frame of reference, is 1.07 x 10^{17} meters.
   a. What is this distance in light years?
   b. What will be the distance to Epsilon Indi as measured by an observer on a rocket heading toward this star at 85% of the speed of light?
   c. How long will it take for this rocket to reach Epsilon Indi according to an observer on the Earth?

3. A rocket is moving toward the star Sirius ["The Dog star"] with a velocity of 2.955 x 10^8 m/s. The distance to Sirius as measured from the rest frame on the Earth is measured to be 8.14 x 10^{16} meters.
   a. What will be the velocity of this rocket as a decimal fraction of the speed of light?
   b. What is the distance to Sirius in light years?
   c. What will be the distance to Sirius as measured by an astronaut on board the rocket?

4. As measured from the Earth the distance to Tau Ceti is determined to be 11.8 c yrs. How fast must a rocket be moving toward Tau Ceti so that the distance to Tau Ceti is reduced to 1.18 c yrs?

5. How fast must a rocket be moving in order for its length to be reduced to 1.0% of its rest length?

6. A rocket is launched toward Barnard's star, which has a rest distance of 6.00 c yrs from the Earth, with an acceleration of 1.20 g's [where 1 g = 9.81 m/s^2].
   a. How long after launch will it take for the speed of the rocket to reach 98% of the speed of light?
   b. Assuming that this rocket travels at a constant speed of 0.980c and then decelerates at -1.2 g's until the rocket reaches Barnard's star. How long will it take for the rocket to arrive at its destination according to an observer at rest on the Earth? [Hint! Try making a graph of velocity vs time!]
   c. What will be the distance to Barnard's star as measured by an astronaut on board the rocket during the constant velocity [0.98c] phase of the trip?

7. The rest diameter of the Milky Way galaxy is measured to be 200,000,000 c yrs as measured by an observer at rest on the Earth. What will be the measured diameter of the Milky Way galaxy as measured by an astronaut on a rocket heading across the across the galaxy at .99999999c?

Answers to opposite side: 8. 1.29 years 9a. 6.12 years b. 1.21 years c. 1.19 c yrs
9d. 1994 AD e. 22.2 years f. 65/minute g. 13/ minute 10. 56 hours 11a. 2014 AD b. 1.93 years
11c. 23.9 years d. 24.1 years 12. 6.23 minutes

PHYSICS HOMEWORK #112 SPECIAL RELATIVITY
TIME DILATION

$$T = T_o\sqrt{1 - V^2 / c^2}$$

WHERE T = time interval measured in the moving system
AND T_o = time interval measured in the rest system

8. According to an observer on the Earth the time it should take to reach a certain star is determined to be 7.50 years. How long will it take to reach this star according to an observer on board a rocket moving toward this star with a velocity of 0.985c?

9. The distance to Barnard's star is measured to be 6.00 c yrs by an observer at rest on the Earth:
   a. Assuming that a rocket is moving toward this star at 0.98c, how long will it take for this rocket to reach Barnard's star according to an observer on the Earth?
   b. How long will it take to reach this star according to an observer on board the rocket?
   c. What will be the distance to Barnard's star according to an observer on board the rocket?
   d. Assuming that the astronaut was 21 years old when she left the Earth is 1988, in what year will the astronaut arrive at Barnard's star?
   e. How old will the astronaut be when she arrives at Barnard's star?
   f. Suppose that the astronaut has a normal heart rate of 65 beats per minute when measured while at rest on the Earth. What will be the astronaut's heart rate as monitored by an observer on board the rocket with the astronaut?
   g. What will the astronaut's heart rate be while on the rocket moving toward Barnard's star, as monitored by an observer on the Earth?

10. Two astronauts play a game of chess on a rocket moving with a velocity of 0.999c away from the Earth. According to the astronauts the game takes 2.5 hours. How long does the game take according to an observer at rest on the Earth?

11. A rocket is moving toward Epsilon Eridani, which is 11.3 c yrs away as measured by an observer at rest on the Earth, at 0.998c. When the astronaut leaves the Earth in 1991 he has just had his 22nd birthday and his young daughter has just turned 1.0 years old. The rocket travels to the star, remains 6.0 months and then returns to the Earth at the same speed:
   a. In what year will the rocket return to the Earth?
   b. How many years will the journey take according to the astronaut on board the rocket?
   c. How old will the astronaut be when he returns to the Earth?
   d. How old will his daughter be when he returns to the Earth?

12. A neutron outside the confines of the nucleus of an atom is unstable and has a life expectancy [half-life] of 6.0minutes. Suppose that a fast moving alpha particle collides with a block of Beryllium and knocks a neutron out of the nucleus with a speed of 8.15×10^7 m/s. What will be the expected lifetime of this neutron as measured by an observer in the rest frame?

Answers to opposite side: 1a. 30.6 meters b. 45.0 meters c. 5.6 meters d. 2.9 c yrs
2a. 11.3 c yrs b. 5.95 c yrs c. 13.3 yrs 3a. 0.985 c b. 8.6 c yrs c. 1.48 c yrs 4. 0.995 c
5. 0.99995 c 6a. 0.79 yrs b. 6.91 yrs c. 1.19 c yrs 7. 28,300 c yrs

PHYSICS HOMEWORK #113 SPECIAL RELATIVITY
VELOCITY TRANSFORMATION

$$v = (v' + V) / (1 + v'V / c^2)$$

WHERE v = velocity measured from rest frame
v' = velocity measure from moving frame
V = velocity of the moving frame

1. A rocket is moving away from the Earth with a velocity of 0.950c. This rocket then launches a missile forward with a velocity of 0.650c relative to the rocket ship. What will be the velocity of this missile as measured from the Earth?

2. A rocket ship is returning from Alpha Centauri with a velocity of 0.970c as measured by an observer on the Earth. At the same time a second rocket ship is moving toward Alpha Centauri with a velocity of 0.930c as measured from the Earth.
   a. What will be the velocity of the first rocket as measured by an observer on board the second rocket?
   b. What will be the velocity of the second rocket as measured by an observer on the first rocket?
   c. What will be the velocity of the first rocket relative to the second rocket as measured by an observer on the Earth?

3. Two rockets are moving toward Tau Ceti in a race. The velocity of rocket A as measured from the Earth is 0.990c while the velocity of rocket B as measured from the Earth is 0.995c.
   a. What will be the velocity of rocket A relative to rocket B as measured by an observer on rocket B?
   b. What will be the velocity of rocket B relative to rocket A as measured by an observer on rocket A?
   c. Assuming that the distance to Tau Ceti is 10.8 c yrs as measured by an observer at rest on the Earth, what will be the distance to Tau Ceti According to an observer on board rocket A?
   d. What will be the distance to Tau Ceti according to the pilot on rocket B?
   e. How long will it take for rocket A to reach Tau Ceti according to the pilot on rocket A?
   f. How long will it take rocket B to reach Tau Ceti according to the pilot on rocket B?
   g. According to the observer on the Earth, how much sooner does rocket B reach Tau Ceti compared to rocket A?

4. Two rockets are on a head on collision course. Rocket A is moving left with a velocity of 0.960c relative to the Earth, while Rocket B is moving right at 0.920c relative to the Earth. Initially, these two rockets are 3.50cyrs apart as measure by an observer on the Earth.
   a. What will be the relative velocity of these two rockets as measure by an observer on either Rocket A or Rocket B?
   b. What is the closing velocity between these two rockets as measured by an observer on the Earth?
   c. How long will it take for these two rockets to meet according to an observer on the Earth?
   d. How long will it take for these two rockets to meet according to an observer on Rocket A?
   e. How long will it take for these two rockets to meet according to an observer on Rocket B?

5. A rocket is moving away from the Earth with a velocity of 0.980c when it fires its forward LASER. What will be the velocity of the LASER light as measured by an observer at rest on the Earth? Explain your answer!

Answers to opposite side: 6. 2.26×10^{9} kg 7. 2.79×10^{-27} kg 8. $\hat{}(1-1.11 \times 10^{-51})/c^2$
9. 8.01×10^{3}kg 10. 0.466c 11. 1.45×10^{-10}N 12. 0.030c 13. 0.99995c 14. 0.9999999c
15. 4.22 Tesla 16a. 1.83×10^{5}N b. 5.84×10^{5}N c. 1.29×10^{9}N

PHYSICS HOMEWORK #114 — SPECIAL RELATIVITY

MASS EXPANSION

$$m = m_o \,/\, \sqrt{1 - V^2 / c^2}$$

WHERE m_o = mass measured while at rest relative to object
and m = mass measured while in motion relative to object

6. A rocket is measured to have a mass of 4.50 x 10^8 kg while at rest on the Earth. What will be the mass of this rocket while moving through space with a velocity of 0.980c as measured by an observer on the Earth?

7. A proton has a rest mass of 1.67 x 10^{-27} kg. What will be the mass of a proton while moving with a velocity of 2.40 x 10^8 m/s?

8. How fast must a proton [m_p = 1.67 x 10^{-27} kg] move in order to have the same mass as a 50.0 gram bullet at rest?

9. A rocket has a rest mass of $25\bar{0}0$ kg. What will be the momentum of this rocket when it is moving with a velocity of 0.95c relative to the observer?

10. A rocket at rest in space ignites its engines. Over a time period of 10.0 seconds the rocket emits $22\bar{0}0$ kg of hot gases out the engine exhaust with a velocity of 0.920c compared to the rest frame. The rest mass of the rocket, including fuel, is $12{,}\bar{0}00$ kg before ignition. Using momentum conservation determine the final velocity of the rocket.

11. A proton is moving in a circular path inside a strong magnetic field generated by a cyclotron with a velocity of 0.980c. What will be the magnitude of the centripetal force required to keep this proton moving in a circular path which has a radius of 5.00 meters?

12. A neutron [mass = 1 AMU] moving with a velocity of 0.990c compared to the rest frame collides with a Uranium 239 [mass = 239 AMU] atom which is at rest. After the collision the two particles stick together. What will be the final velocity of this combination? [Use momentum conservation but note that it is NOT necessary to treat the mass of the final combination relativistically! Why not?]

13. What velocity would be required to increase the mass of an object by a factor of $10\bar{0}$x?

14. How fast must an electron mass move in order to have the same mass as a proton at rest?

15. A proton is moving with a velocity of 2.90 x 10^8 m/s when it enters a uniform magnetic field. Upon entering the magnetic field the proton begins to move in a circular path which has a radius of 2.80 meters. What must the strength of the magnetic field be in order to keep this proton moving in this circular path?

16. Consider a rocket ship which has a rest mass of 18,600 kg. and is accelerating through space at 9.80 m/s^2.
 a. How much force would be required to accelerate this rocket if the velocity of the rocket is 0.100c?
 b. How much force would be required to accelerate this rocket if the velocity of the rocket is 0.950c?
 c. How much force would be required to accelerate this rocket if the velocity of the rocket is $0.99999\bar{9}99$c?

Answers to opposite side: 1. 0.989c 2a. 0.999c b. 0.999c c. 1.90c 3a. -0.334c
3b. 0.334c 3c. 1.52 cyrs d. 1.08 cyrs e. 1.54 yrs f. 1.09 yrs g. 0.055 yrs 4a. 0.998c
4b. 1.88c c. 1.86 yrs d. 0.52 yrs e. 0.73 yrs 5. c

PHYSICS HOMEWORK #115 SPECIAL RELATIVITY
ENERGY RELATIONSHIPS

$RE = m_o c^2$ $TE = mc^2$ $KE = TE - RE = mc^2 - m_o c^2 = m_o c^2 (\beta - 1)$

1. What will be the rest energy of a 2.50 kg mass at rest?
2. What is the rest energy of a proton?
3. Suppose that an electron moving with a non_relativistic speed encounters a positron moving at an identical but opposite non relativistic speed. These two particles meet and then each particle disappears and in their place remains only energy.
   a. How much energy will be released by this interaction?
   b. Assuming that the energy which is released escapes in the form of two identical light photons, what will be the wavelength of each photon?
   c. Why must the released energy take the form of TWO photons?
4. What will be the total energy of a 2.50 kg mass moving with a velocity of 0.980c?
5. What will be the total energy of a proton moving with a velocity of 0.990c?
6. What will be the kinetic energy of a proton moving with a velocity of 0.990c?
7. A rocket is moving toward Alpha Centauri with a velocity of 0.995c:
   a. What will be the rest energy of this rocket if its rest mass is $76,\bar{0}00$ kg.?
   b. What will be the total energy of this rocket?
   c. What will be the kinetic energy of this rocket?
   d. By what percentage would the value for the kinetic energy have been wrong had you used the classical equation for the calculation of the kinetic energy instead of the relativistic equation?

8. An electron moving with a velocity of 0.995c collides with a heavy atom. As a result of this collision the speed of the electron is reduced to 0.450c. What will be the energy lost by the electron in this collision?

9. The atomic mass of a single mole of Carbon 12 is defined to be exactly 12.000 grams.
   a. What is the mass of a single atom of Carbon 12?
   b. A single atom of Carbon 12 consists of 6 protons, 6 neutrons and 6 electrons. What will be the total mass of a Carbon atom based on the sum of its parts [carry out your calculations to at least 5 significant digits!]
   c. What is the nuclear mass defect of a single atom of Carbon 12?
   d. How much energy will be released from the "construction" of 1.00 mole of Carbon 12?
   e. If mass is supposed to be conserved, where did the missing mass go?
   f. Assuming that all of the released energy remains with the resulting Carbon atom, determine the velocity of the resulting atom.

REST MASSES

Proton = 1.6726×10^{-27} kg = 1.00728 AMU
Neutron = 1.6750×10^{-27} = 1.008665 AMU
Electron = 9.11×10^{-31} kg = 0.000549 AMU
1 AMU = 1.6606×10^{-27} kg = mass of C_{12}

Answers to opposite side: 1. 30.0 meters 2. 135 meters 3. 420,000 kg 4. 1,890,000 kg 5. 0.955 c yrs 6. 4.41 years 7. 0.979 years 8. 1.7×10^{23} Joules 9. 3.8×10^{22} Joules 10. 1.3×10^{23} J 11. 28 years 12. 2003 AD 13. 0.999970c 14. 0.99960c 15. 5.5×10^{14} kg m/s 16a. c b. 0.999980c 16c. depends on the observer!

PHYSICS HOMEWORK #116 SPECIAL RELATIVITY
ALL TOGETHER

A rocket, which has a rest mass of 42$\bar{0}$,000 kg and a rest length of 135 meters, is moving toward Alpha Centauri with a velocity of 0.975c. The distance to Alpha Centauri, as measured from an observer at rest on the Earth, is 4.30 cyrs.

1. What will be the length of this rocket as measured by an observer at rest on the Earth?
2. What will be the length of the rocket as measured by an observer on board the rocket?
3. What will be the mass of the rocket as measured by an observer on board the rocket?
4. What will be the mass of the rocket as measured by an observer at rest on the Earth?
5. What will be the distance to Alpha Centauri as measured by an observer on board the rocket?
6. How long will it take for the rocket to reach Alpha Centauri according to an observer at rest on the Earth?
7. How long will it take for the rocket to reach Alpha Centauri according to an observer on board the rocket?
8. What will be the total energy of this rocket as it moves toward Alpha Centauri as measured by an observer at rest on the Earth?
9. What is the rest energy of this rocket?
10. What will be the kinetic energy of this rocket as measured by an observer at rest on the Earth?
11. If the astronaut on board the rocket was 24.0 years old when she left the Earth, spent exactly 2.00 years on a planet orbiting Alpha Centauri, and then returned to Earth at the same speed. How old will the astronaut be when she returns to the Earth?
12. Assuming that this astronaut left the Earth in 1992, in what year will she return to the Earth?
13. Assuming that this rocket on the return trip encounters a second rocket on its way to Alpha Centauri with a velocity of 0.998c, as measured form the Earth. What will be the velocity of the second rocket as measured by an observer on board the first rocket?
14. What should the velocity of a rocket be if it is to traverse the distance to Alpha Centauri in only 32.0 days rocket time?
15. What is the momentum of the first rocket as measured by an observer at rest on the Earth?
16. Suppose that the rocket returning from Alpha Centauri suddenly discovers that the second rocket on its way to Alpha Centauri is a space pirate. The space pirate fires a LASER blast [light] at the first rocket and the first rocket responds by firing a photon torpedo at the pirate with a velocity of 0.870c relative to the first rocket.
    a. What will be the velocity of the LASER blast as measured by the first rocket?
    b. What will be the velocity of the photon torpedo as measured by the pirate rocket?
    c. Assuming that these two rockets were 0.100 c yrs apart, as measured by an observer at rest on the Earth, when the two weapons were fired, how long does each have to live in [in their own reference frames!]?

Answers to opposite side: 1. 2.3 x 10^{17} Joules 2. 1.5 x 10^{-10} Joules 3a. 1.6 x 10^{-13} Joules 3b. 0.0024nm 4. 1.13 x 10^{18} Joules 5. 1.1 x 10^{-9} Joules 6. 9.2 x 10^{-10} Joules 7a. 6.8 x 10^{21} Joules 7b. 6.9 x 10^{22} Joules 7c. 6.2 x 10^{22} Joules d. -94% 8. 7.3 x 10^{-13} Joules 9a. 1.9924 x 10^{-26} kg 9b. 2.0091 x 10^{-26} kg 9c. 1.67 x 10^{-28} kg d. 9.05 x 10^{12} Joules e. KE f. 0.999985c