

MOMENT OF INERTIA - INTEGRATION

Moment of Inertia through Integration

- Consider a disk which has a radius of $R = 12.0$ cm and which has a mass density σ which varies with the radius of the disk according to the function $\sigma = (18.0 + 36.0 r^2)$ kg/m².
 - What is the total mass of this disk?
 - What will be the moment of inertia of this disk through an axis through the center of mass and perpendicular to the plane of the disk?
- Consider a rod which has a length of $L = 56.0$ cm and which has a mass density which varies with length according to the formula $\lambda = (28.0 + 44.0r^2)$ kg.
 - What is the total mass of this rod?
 - Where is the center of mass of this rod?
 - What is the moment of inertia of this rod about an axis perpendicular to the rod and through the lighter end of the rod?
- Consider a rectangular plate $x = 35.0$ cm long and $y = 15.0$ cm wide, as shown to the right, where the surface mass density of the plate varies with the x coordinate according to $\sigma = (120 - 2800 x^3)$ kg/m².
 - What is the total mass of this plate?
 - Where is the center of mass of this rectangular plate?
 - What will be the moment of inertia of this plate about an axis along the left edge of the plate?
 - What will be the moment of inertia of this plate about the bottom edge of the rectangular plate?



- Consider a thick ring with an inner radius of $r_1 = 6.00$ cm and an outer radius of $r_2 = 16.0$ cm where the mass per unit area varies according to $\sigma = (85.0 + 25.0 r^2)$ kg/m².
 - what is the total mass of this ring?
 - What will be the moment of inertia of this ring through an axis perpendicular to the plane of the ring and through the center of mass?

Moment of Inertia - Parallel axis Theorem/Plane Figure Theorem

- Consider a uniform disk which has a radius of $R = 35.0$ cm and a total mass of $M = 8.20$ kg.
 - What is the moment of inertia of this disk about an axis perpendicular to the plane of the disk and through the center of mass?
 - What will be the moment of inertia of this disk about an axis perpendicular to the plane of the disk and through the outer edge of the disk?
 - What will be the moment of inertia of this disk about an axis in the plane of the disk and through the center of mass of the disk?
 - What will be the moment of inertia of this disk about an axis in the plane of the disk and tangent to the edge of the disk?
 - What will be the moment of inertia of this disk about an axis perpendicular to the plane of the disk and through a point 155 cm from the center of mass?
- Consider a rectangular plate which has a length of $L = 35.0$ cm., a width of $w = 20.0$ cm. and a uniformly distributed mass of $m = 6.60$ kg.
 - What is the moment of inertia of this plate about an axis in the plane of the plate and tangent to the width of the plate?
 - What will be the moment of inertia of this plate about an axis in the plane of the plate and tangent to the length of the plate?

Answers to opposite side: 6c. 0.358 kgm^2 d. 0.0894 kgm^2 e. 0.0674 kgm^2 f. 0.0220 kgm^2 g. 0.0894 kgm^2
 7a. 0.847 kgm^2 b. 2.96 kgm^2 c. 27.8 kgm^2 8a. 173 kg b. $0.583, 0.110 \text{ m}$ c. 64.8 kgm^2 d. 2.78 kgm^2
 8e. 67.6 kgm^2 f. 6.84 kgm^2 g. 0.696 kgm^2 h. 6.15 kgm^2 9a. 3.11 kg b. 0.0594 kgm^2 c. 0.0297 kgm^2
 9d. 0.131 kgm^2 e. 0.160 kgm^2

PHYSICS HOMEWORK #68

ROTATIONAL KINEMATICS

MOMENT OF INERTIA - PLANE FIGURE/PARALLEL AXIS THEOREMS

- 6c. What will be the moment of inertia of this plate about an axis perpendicular to the plane of the plate and through one of the corners of the plate?
- d. What will be the moment of inertia of this plate about an axis perpendicular to the plate and through the center of mass of the plate?
- e. What will be the moment of inertia of this plate about an axis in the plane of the plate and through the center of mass parallel to the width of the plate?
- f. What will be the moment of inertia of this plate about an axis parallel to the plane of the plate, through the center of mass and parallel to the length L of the plate?
- g. What will be the moment of inertia about an axis perpendicular to the plane of the plate and through the center of mass of the plate? [Use your answers to e & f above to determine this moment of inertia!]
7. Consider a uniform sphere which has a radius of $R = 42.0$ cm and a total mass of $M = 12.0$ kg.
- a. What is the moment of inertia of this sphere about an axis through the center of mass?
- b. What will be the moment of inertia of this sphere about an axis tangent to the surface of the sphere?
- c. What will be the moment of inertia of this sphere about an axis tangent to the surface of the sphere but at a distance of 150 cm from the center of the sphere?
8. Consider a rectangular plate $x = 82.0$ cm long and $y = 22.0$ cm wide, as shown to the right, where the surface mass density of the plate varies with the x coordinate according to $\sigma = (15\bar{0} + 36\bar{00} x^2)$ kg/m².
- a. What is the total mass of this plate?
- b. What are the x and y coordinates of the center of mass of this rectangular plate?
- c. What will be the moment of inertia of this plate about an axis along the left edge of the plate?
- d. What will be the moment of inertia of this plate about the bottom edge of the rectangular plate?
- e. What will be the moment of inertia about an axis perpendicular to the plane of the plate and through the bottom left corner of the plate as shown in the diagram?
- f. What will be the moment of inertia of this plate about an axis perpendicular to the plane of the plate and through the center of mass?
- g. What will be the moment of inertia about an axis in the plane of the plate, through the center of mass and parallel to the x axis?
- h. What will be the moment of inertia about an axis in the plane of the plate, through the center of mass and parallel to the y axis? [Confirm that you can get this answer through both the parallel axis theorem and the plane figure theorem!]
9. Consider a disk which has a radius of $R = 18.0$ cm and which has a mass density σ which varies with the radius of the disk according to the function $\sigma = (18.0 + 54\bar{00} r^3)$ kg/m².
- a. What is the total mass of this disk?
- b. What will be the moment of inertia of this disk about an axis through the center of mass and perpendicular to the plane of the disk?
- c. What will be the moment of inertia of this disk about an axis parallel to the plane of the disk and through the center of mass?
- d. What will be the moment of inertia of this disk about an axis parallel to the plane of the disk and tangent to the edge of the disk?
- e. What will be the moment of inertia of this disk about an axis perpendicular to the plane of the disk and through the edge of the disk?



Answers to opposite side: 1a. 0.826 kg b. 0.00598 kgm² 2a. 18.3 kg b. 0.300 m c. 2.12 kgm²
3a. 4.72 kg b. 0.140, 0.075 m c. 0.129 kgm² d. 0.0354 kgm² 4a. 5.90 kg b. 0.0862 kgm²
5a. 0.502 kgm² b. 1.507 kgm² c. 0.251 kgm² d. 1.26 kgm² e. 20.2 kgm²
6a. 0.270 kgm² b. 0.0880 kgm²