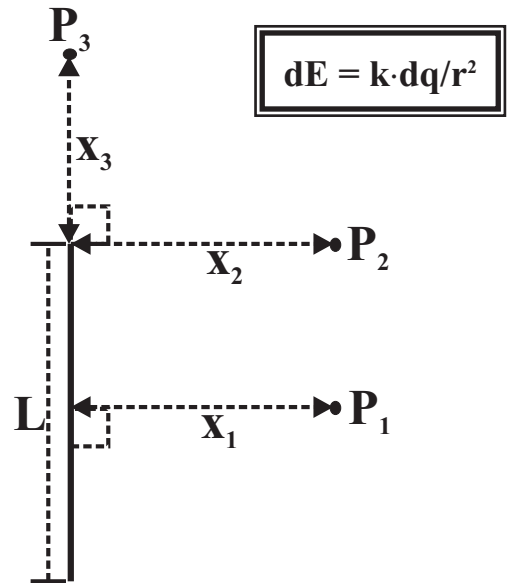


**PHYSICS HOMEWORK #135**

**ELECTROSTATICS**

**ELECTRIC FIELDS - CONTINUOUS CHARGE DISTRIBUTIONS**

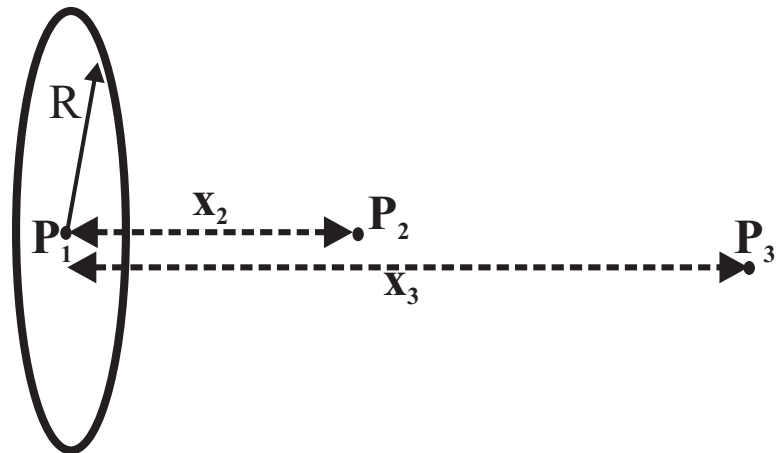
1. A charge of  $q = 60.0 \mu\text{C}$  is distributed uniformly along a wire which is  $L = 30.0 \text{ cm}$  long. Point  $P_1$  is located  $x_1 = 5.00 \text{ cm}$  from the center of the wire, point  $P_2$  is located  $x_2 = 5.00 \text{ cm}$  from the end of the wire and point  $P_3$  is located  $x_3 = 5.00 \text{ cm}$  from the end of the wire.
- What is the charge per unit length  $\lambda$  of this wire?
  - What will be the x and y components of the electric field  $E_1$  at point  $P_1$ ?
  - What will be the x and y components of the electric field  $E_2$  at point  $P_2$ ?
  - What will be the x and y components of the electric field at  $E_3$  point  $P_3$ ?



**Suppose that instead of the charge being uniformly distributed along the length of the wire, there is a charge distribution according to  $\lambda = (150 + 300y)\mu\text{C/m}$ . Assume that  $y = 0$  at the bottom end of the wire.**

- What will be the total charge  $q$  contained on this wire?
- What will be the x and y components of the electric field  $E_1$  at point  $P_1$ ?
- What will be the x and y components of the electric field at  $E_2$  point  $P_2$ ?
- What will be the x and y components of the electric field at  $E_3$  point  $P_3$ ?

2. A charge of  $q = 80.0 \mu\text{C}$  is distributed uniformly around a circular loop of wire which has a radius of  $R = 5.00 \text{ cm}$ . Point  $P_1$  is located at the center of the loop,  $P_2$  is located  $x_2 = 12.0 \text{ cm}$  from the center of the loop and  $P_3$  is located  $x_3 = 8.25 \text{ m}$  from the center of the loop.

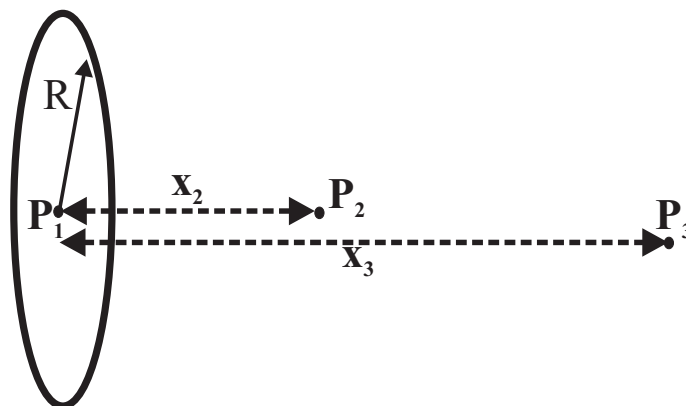


- What will be the charge density per unit length  $\lambda$  for this loop?
- What will be the x and y components of the electric field  $E_1$  at point  $P_1$ ?
- What will be the x and y components of the electric field  $E_2$  at point  $P_2$ ?
- What will be the x and y components of the electric field  $E_3$  at point  $P_3$ ?

Answers to opposite side: 3a.  $1.70 \times 10^3 \mu\text{C/m}^2$  b.  $9.59 \times 10^7 \text{ N/Ci}$  3c.  $3.60 \times 10^7 \text{ N/Ci}$  d.  $1.59 \times 10^4 \text{ N/Ci}$   
 3e.  $112 \mu\text{C}$  f.  $8.48 \times 10^7 \text{ N/Ci}$  g.  $3.33 \times 10^7 \text{ N/Ci}$  h.  $1.48 \times 10^4 \text{ N/Ci}$  4a.  $424 \mu\text{C/m}^2$  b.  $1.33 \times 10^7 \text{ N/Ci}$   
 4c.  $1.07 \times 10^7 \text{ N/Ci}$  d.  $108 \text{ N/Ci}$

ELECTRIC FIELDS - CONTINUOUS CHARGE DISTRIBUTIONS

3. A charge of  $q = 120\mu\text{C}$  is distributed uniformly across the surface of a solid, insulating disk which has a radius of  $R = 15.0\text{cm}$ . Point  $P_1$  is located a distance of  $0.0100\text{cm}$  from the center of the disk,  $P_2$  is located  $x_2 = 12.0\text{cm}$  from the center of the disk and  $P_3$  is located  $x_3 = 8.25\text{m}$  from the center of the disk.

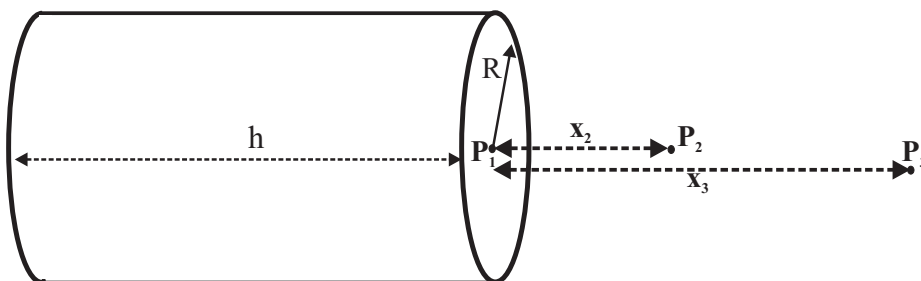


- What will be the surface charge density  $\sigma$  on this disk?
- What will be the x and y components of the electric field  $E_1$  at point  $P_1$ ?
- What will be the x and y components of the electric field  $E_2$  at point  $P_2$ ?
- What will be the x and y components of the electric field  $E_3$  at point  $P_3$ ?

Suppose that instead of the charge being uniformly distributed across the disk, there is a charge distribution according to  $\sigma = (1500 + 800r)\mu\text{C}/\text{m}^2$ .

- What will be the total charge  $q$  on the surface of this disk?
- What will be the x and y components of the electric field  $E_1$  at point  $P_1$ ?
- What will be the x and y components of the electric field  $E_2$  at point  $P_2$ ?
- What will be the x and y components of the electric field  $E_3$  at point  $P_3$ ?

4. A charge of  $q = 120\mu\text{C}$  is distributed uniformly across the surface of an insulating cylinder which has a radius of  $R=15.0\text{cm}$  and a length of  $30.0\text{cm}$ . A point  $P_1$  is located at the end of the cylinder, a point  $P_2$  is located  $x_2 = 12.0\text{cm}$  from the end of the cylinder while a point  $P_3$  is located  $x_3 = 100\text{m}$  from the end of the cylinder.



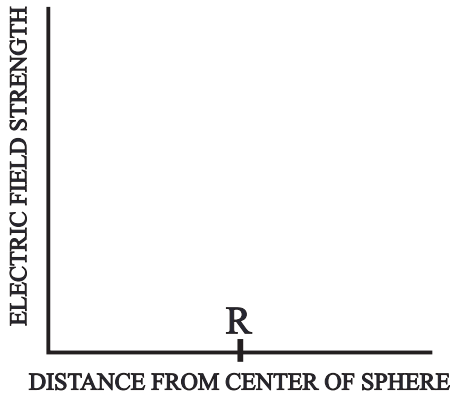
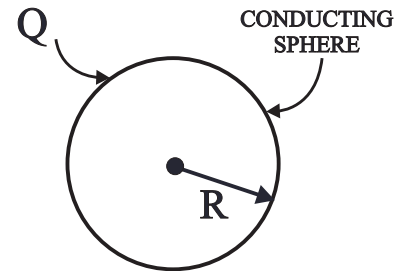
- What will be the charge density  $\sigma$  on the surface of this insulating cylinder?
- What will be the x and y components of the electric field at point  $P_1$ ?
- What will be the x and y components of the electric field at point  $P_2$ ?
- What will be the x and y components of the electric field at point  $P_3$ ?

Answers to opposite side: 1a.  $200\mu\text{C}/\text{m}$  b.  $6.83 \times 10^7\text{N}/\text{Ci}$ ,  $0.0\text{N}/\text{Cj}$  c.  $3.55 \times 10^7\text{N}/\text{Ci}$ ,  $3.01 \times 10^7\text{N}/\text{Cj}$   
 1d.  $3.09 \times 10^7\text{N}/\text{C}$  1e.  $58.5\mu\text{C}$  f.  $6.66 \times 10^7\text{N}/\text{Ci}$ ,  $-4.70 \times 10^6\text{N}/\text{Cj}$  g.  $4.04 \times 10^7\text{N}/\text{Ci}$ ,  $3.20 \times 10^7\text{N}/\text{Cj}$   
 1h.  $3.41 \times 10^7\text{N}/\text{Cj}$  2a.  $255\mu\text{C}/\text{m}$  2b.  $0.0\text{N}/\text{C}$  c.  $3.93 \times 10^7\text{N}/\text{Ci}$ ,  $0.0\text{N}/\text{Cj}$  d.  $1.06 \times 10^4\text{N}/\text{Ci}$ ,  $0.0\text{N}/\text{Cj}$

ELECTRIC FIELDS - GAUSS' LAW

1. A charge of  $Q = +25.0 \mu\text{C}$  is distributed evenly on the surface of a conducting sphere, which has a radius of  $R = 5.00\text{cm}$ .
  - a. What will be the charge density  $[\sigma]$  on this surface?
  - b. What should be the shape of a Gaussian surface such that all points on this surface have the same electric field strength and so that the electric field is perpendicular to all points on the surface? Why are these conditions desirable?
  - c. What will be the electric field strength  $3.00\text{cm}$  from the center of this sphere?
  - d. Using Gauss's Law develop an expression which will predict the electric field strength for any point outside this sphere.  $[r > R]$
  - e. What will be the electric field strength  $7.00\text{cm}$  from the center of this sphere?
  - f. On the diagram to the right sketch the electric field in the vicinity of this charged sphere.
  - g. On the graph below sketch the electric field strength as a function of distance from the center of the sphere.

$$\int E \cdot dA = 4 \cdot \pi \cdot k \cdot q_{\text{enclosed}}$$



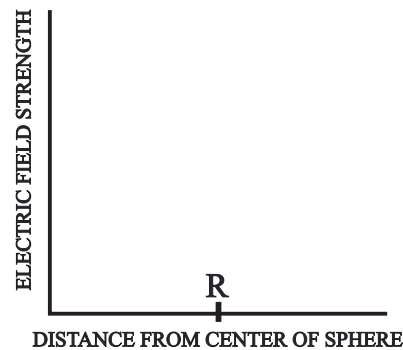
2. A charge of  $Q = +45.0 \mu\text{C}$  is distributed uniformly across the surface of an insulating plane [the thickness is insignificant] which is  $35.0 \text{ cm}$  wide and  $55.0 \text{ cm}$  high.
  - a. What will be the surface charge density  $\sigma$  on this plane?
  - b. What should be the shape of a Gaussian surface such that all points on this surface have the same electric field strength and so that the electric field is perpendicular to all points on the surface? Explain!
  - c. Using Gauss's Law develop an expression predicting the electric field strength as a function of distance from this finite plane.
  - d. What will be the electric field strength  $R = 2.00 \text{ cm}$  from the center of this finite plane?
  - e. What will be the electric field strength  $R = 5.00 \text{ cm}$  from the center of this plane?
  - f. In general, how does the electric field strength vary with increasing distance from a uniform, planar distribution of charge?
  - g. What will be the electric field strength  $2,550 \text{ meters}$  from the center of this plane?
  - h. Why is the answer to **g** different from the answer to **d**? Explain!

3. A charge of  $-15.0 \mu\text{C}$  is distributed evenly along a conducting wire which is  $70.0 \text{ cm}$  long.
  - a. What will be the charge density  $\lambda$  along this wire?
  - b. Using Gauss's Law, develop an expression predicting the electric field strength as a function of distance from the charged wire.
  - c. What will be the strength of the electric field  $R = 2.00 \text{ cm}$  from the center of this wire?
  - d. In general, how does the electric field strength vary with distance from a charged, straight wire? Explain!
  - e. What will be the electric field strength  $1250 \text{ meters}$  from the center of this wire?
  - f. Why is the answer to **e** different from **c**? Explain!
  - g. What general statement can you make about the electric field strength very far away from any **FINITE** charged body? Explain!

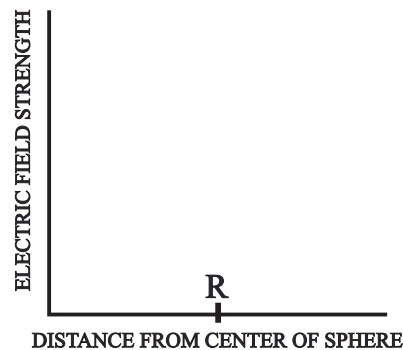
Answers to opposite side: 4a.  $1.04 \times 10^{-2} \text{ C/m}^2$  b. sphere c.  $\frac{4}{3} \pi r^3 \rho$  d.  $1.18 \times 10^{-6} \text{ C}$   
 4e.  $\frac{4}{3} \pi k r \rho$  f.  $1.57 \times 10^7 \text{ N/C}$  g.  $q = Q$  h.  $+15 \mu\text{C}$  i.  $kQ/r^2$  j.  $7.99 \times 10^6 \text{ N/C}$  5a.  $-1.11 \times 10^{-2} \text{ C/m}^2$   
 5b. cylinder c.  $\pi r^2 L \rho$  d.  $-8.75 \times 10^{-7} \text{ C}$  e.  $q = Q$  f.  $-3.50 \mu\text{C}$  g.  $2\pi k r \rho$  h.  $6.30 \times 10^6 \text{ N/C}$  i.  $2kQ/Lr$   
 5j.  $6.3 \times 10^6 \text{ N/C}$

ELECTRIC FIELDS - GAUSS' LAW

4. A solid, insulating sphere, which has a radius of  $R = 7.00$  cm, contains  $Q = +15.0 \mu\text{C}$  of charge distributed uniformly throughout its interior.
- What will be the charge density  $\rho$  contained within this sphere?
  - What should be the shape of a Gaussian surface such that all points on this surface have the same electric field strength and so that the electric field is perpendicular to all points on the surface?
  - Develop an expression for the charge contained within any Gaussian surface for which the radius of the Gaussian surface  $r$  is less than the radius of the solid sphere. [i.e.  $r < R$ ]
  - How much charge will be contained within a Gaussian surface which has a radius of  $r = 3.00$  cm?
  - Using Gauss's Law develop an expression for the electric field strength anywhere within this solid sphere.
  - What will be the electric field strength  $r = 4.00$  cm from the center of this solid sphere?
  - Develop an expression for the total charge contained within any Gaussian surface where the radius of the Gaussian surface  $r$  is greater than the radius of the solid sphere  $R$ .
  - How much charge will be contained within a Gaussian surface which has a radius of  $r = 13.0$  cm?
  - Develop an expression describing the electric field strength anywhere outside this solid sphere. [i.e.  $r > R$ ]
  - What will be the electric field strength  $r = 13.0$  cm from the center of this solid sphere?
  - On the graph at the right sketch the electric field strength as a function of distance from the center of the solid sphere.



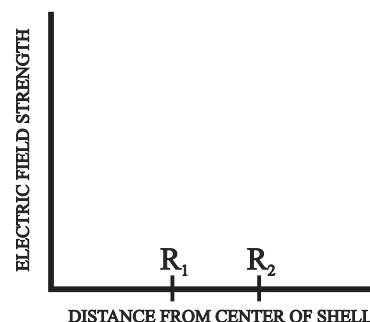
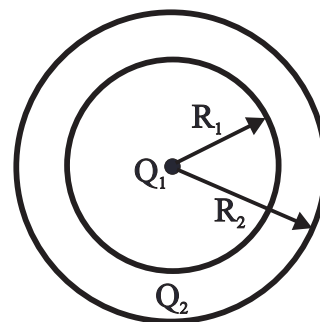
5. Consider an insulating cylinder, which is  $L = 25.0$  cm long, has a radius of  $R = 2.00$  cm and contains  $Q = -3.50 \mu\text{C}$  of charge distributed uniformly throughout its interior.
- What will be the charge density  $\rho$  contained within this cylinder?
  - What should be the shape of the Gaussian surface for this solid cylinder so that the electric field strength is uniform at all points on the Gaussian surface and such that the electric field is perpendicular to all points on the Gaussian surface?
  - Develop an expression describing the charge contained within a Gaussian surface which has a radius  $r$  that is less than the radius  $R$  of the cylinder.
  - How much charge will be contained within a Gaussian surface which has a radius of  $r = 1.00$  cm?
  - Develop an expression which will predict the charge contained within a Gaussian surface which has a radius  $r$  that is greater than the radius  $R$  of the solid cylinder.
  - How much charge will be contained within a Gaussian surface which has a radius of  $r = 4.00$  cm?
  - Develop an expression which will predict the electric field strength as a function of distance from the center of the solid cylinder for distances  $r$  less than the radius  $R$  of the cylinder.
  - What will be the electric field strength  $r = 1.00$  cm from the center of this cylinder?
  - Develop an expression describing the electric field strength for points outside of the solid cylinder. [ $r > R$ ]
  - What will be the electric field strength  $r = 4.00$  cm from the center of the solid cylinder?
  - On the graph at the right plot the electric field as a function of distance from the center of the cylinder.



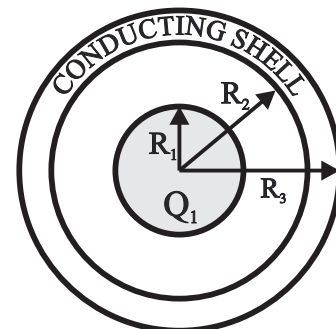
Answers to opposite side: 1a.  $7.96 \times 10^{-4} \text{ C/m}^2$     b. sphere    c. zero    d.  $kQ/r^2$     e.  $4.59 \times 10^7 \text{ N/C}$   
 2a.  $2.34 \times 10^{-4} \text{ C/m}^2$     b. "tin can"    c.  $2\pi k\sigma$     d.  $1.32 \times 10^7 \text{ N/C}$     e. same as d    f.  $E = \text{constant}$   
 2g. far away a finite plane looks like a point.    3a.  $2.14 \times 10^{-5} \text{ C/m}$     b.  $2kQ/Lr$     c.  $1.93 \times 10^7 \text{ N/C}$     d. inversely  
 3e.  $8.64 \times 10^{-2} \text{ N/C}$     f. far away a finite line looks like a point.    g. far enough away any finite object looks like a point!

ELECTRIC FIELDS - GAUSS' LAW

6. Consider a conducting spherical shell, which has an inner radius of  $R_1 = 7.00$  cm, an outer radius of  $R_2 = 9.00$  cm and contains a charge of  $Q_2 = +18.0 \mu\text{C}$ . This shell, in turn, encloses a point charge of  $Q_1 = -6.00 \mu\text{C}$  located at its center, as shown in the diagram at the right.
- Using Gauss's Law develop an expression which will predict the electric field strength inside the spherical shell [for values of  $r < R_1$ ] and evaluate that expression for a value of  $r = 5.00$  cm.
  - Using Gauss's Law develop an expression which will predict the electric field strength within the spherical shell and evaluate that expression for a point  $r = 8.00$  cm from the center of the shell.
  - Using Gauss's Law develop an expression which will predict the electric field strength outside of the spherical shell and evaluate that expression for a value of  $r = 12.0$  cm.
  - What will be the charge density on the inner surface of the spherical shell?
  - What will be the charge density on the outer surface of the spherical shell?
  - On the diagram to the right sketch the electric field [lines of force] everywhere.
  - On the graph to the right sketch the electric field strength as a function of distance from the center of the spherical shell.



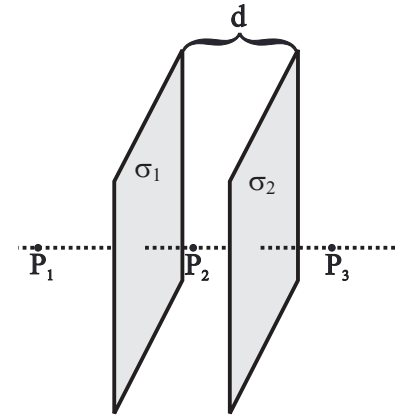
7. An insulating, solid sphere has a radius of  $R_1 = 3.50$  cm and contains a charge of  $55.0 \mu\text{C}$  which is distributed uniformly throughout its volume. This sphere is in turn surrounded by an uncharged, conducting spherical shell, which has an inner radius of  $R_2 = 6.50$  cm and an outer radius of  $R_3 = 8.00$  cm.
- Using Gauss's Law develop an expression which will predict the electric field strength everywhere inside of the solid sphere and evaluate that expression for a value of  $r = 1.0$  cm. [ $r < R_1$ ]
  - Using Gauss's Law develop an expression which will predict the electric field strength outside of the solid sphere but inside the spherical shell and evaluate that expression for a value of  $r = 4.00$  cm. [ $R_1 < r < R_2$ ]
  - What will be the electric field strength within the spherical shell? Justify! [ $R_2 < r < R_3$ ]
  - Using Gauss's Law develop an expression which will predict the electric field strength everywhere outside of the spherical shell and evaluate that expression for a value of  $r = 10.5$  cm. [ $r > R_3$ ]
  - What will be the surface charge density  $\sigma_2$  on the inner surface of the spherical shell?
  - What will be the surface charge density  $\sigma_3$  on the outer surface of the spherical shell?
- Suppose, instead, that the insulating, charged sphere has a charge density which varies as a function of distance from the center of the sphere according to the relationship  $\rho = (4.17 \times 10^4 + 1.91 \times 10^7 r^2) \mu\text{C}/\text{m}^3$
- What will be the total charge  $Q_1$  of this insulating, charged sphere?
  - What will be the strength of the electric field  $r = 2.0$  cm from the center of this insulating sphere?
  - What will be the strength of the electric field  $r = 5.0$  cm from the center of the insulating sphere?



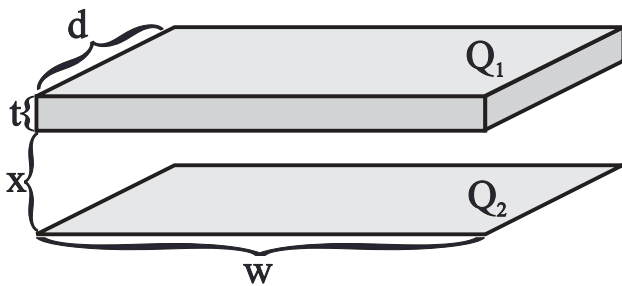
Answers to opposite side: 8a.  $7.07 \times 10^4$  N/C i b.  $2.12 \times 10^5$  N/C c.  $-7.07 \times 10^4$  N/C i  
 9a.  $-3.32 \times 10^7$  N/C j b.  $1.81 \times 10^7$  N/C j c.  $-1.51 \times 10^7$  N/C j d.  $-2.41 \times 10^{-12}$  N j e.  $5.13 \times 10^7$  N/C j  
 f.  $-5.13 \times 10^7$  N/C j g. zero h.  $1.33 \times 10^{-4}$  C/m<sup>2</sup>,  $4.53 \times 10^{-4}$  C/m<sup>2</sup> 10a. zero for [ $r < R_1$ ],  $2k\lambda/r$  for [ $R_1 < r < R_2$ ],  
 $2k(\lambda + \rho[r^2 - R_2^2])\pi/r$  for [ $R_2 < r < R_3$ ],  $2\pi k(\lambda_1 + \lambda_2)/r$  for [ $r > R_3$ ] c.  $0.0$  N/C,  $2.35 \times 10^6$  N/C,  $1.72 \times 10^6$  N/C

ELECTRIC FIELDS - GAUSS' LAW

8. Consider two vertically oriented, parallel infinite planes. The left hand plane has a charge density of  $\sigma_1 = 1.25 \mu\text{C}/\text{m}^2$  while the right hand plane has a charge density of  $\sigma_2 = -2.50 \mu\text{C}/\text{m}^2$ . The two planes are  $d = 5.00 \text{ cm}$  apart as shown to the right.
- Using Gauss's Law, determine the electric field strength to the left of the two planes at point  $P_1$ .
  - Using Gauss's Law, determine the electric field strength between the two planes at point  $P_2$ .
  - Using Gauss's Law, determine the electric field strength to the right of the two planes at point  $P_3$ .

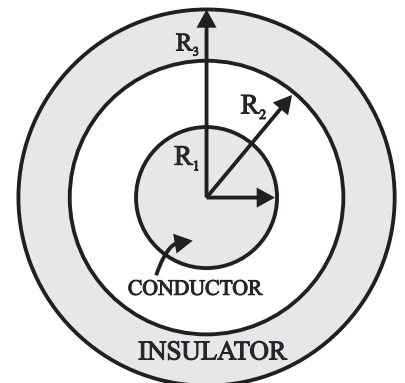


9. Consider two horizontally oriented, parallel plates as shown below. Each plate has a width of  $w = 25.0\text{cm}$  and a depth of  $d = 15.0\text{cm}$ . The upper plate is a conductor which has a thickness of  $t = 1.00\text{cm}$ , while the lower plate is an insulator of negligible thickness. These two plates are a distance of  $x = 5.00 \text{ cm}$  apart. A total charge of  $Q_1 = 22.0 \mu\text{C}$  is placed on the upper plate, while a total charge of  $Q_2 = +12.0 \mu\text{C}$  is placed on the lower plate.



- What will be the direction and magnitude of the electric field between these two planes as caused by the upper plane?
  - What will be the direction and magnitude of the electric field between these two planes as caused by the lower plane?
  - What will be the direction and magnitude of the electric field between these two planes as caused by both planes?
  - What will be the direction and magnitude of the electrostatic force acting on a proton placed in the area between these two plates?
  - What will be the direction and magnitude of the electric field in the area above these two planes?
  - What will be the direction and magnitude of the electric field in the area below these two planes?
- What will be the direction and magnitude of the electric field within the upper plate?
  - What will be the charge densities  $\sigma_1$  and  $\sigma_1'$  on the upper and lower surfaces of the upper plate?
  - Make a sketch showing the electric field everywhere in the vicinity of these plates.

10. Consider a conducting cylinder which has a radius of  $R_1 = .0500\text{cm}$  and which contains a charge density of  $\lambda_1 = +2.20\mu\text{C}/\text{m}$  of length and which is surrounded by an insulating cylindrical shell which has an inner radius of  $R_2 = 1.50\text{cm}$  and an outer radius of  $R_3 = 2.25\text{cm}$ . This outer cylinder has charge distributed uniformly throughout with a charge density of  $\rho_2 = +747 \mu\text{C}/\text{m}^3$ .
- Develop the set of expressions that will predict the electric field everywhere in the area of these two cylinders.
  - Make a graph showing the electric field strength as a function of distance from the center of the cylinders.
  - What will be the electric field strength  $1.0\text{cm}$ ,  $2.0 \text{ cm}$  and  $3.0\text{cm}$  from the center of the cylinders.



Answers to opposite side: 6a.  $2.16 \times 10^7 \text{ N/C}$    b. zero   c.  $7.5 \times 10^6 \text{ N/C}$    d.  $9.74 \times 10^{-5} \text{ C/m}^2$   
 6e.  $1.18 \times 10^{-4} \text{ C/m}^2$    7a.  $1.15 \times 10^8 \text{ N/C}$    b.  $3.09 \times 10^8 \text{ N/C}$    c. zero   d.  $4.49 \times 10^7 \text{ N/C}$    e.  $-1.04 \times 10^{-3} \text{ C/m}^2$   
 7f.  $6.84 \times 10^{-4} \text{ C/m}^2$    g.  $10.0 \mu\text{C}$    h.  $3.49 \times 10^7 \text{ N/C}$    i.  $3.6 \times 10^7 \text{ N/C}$