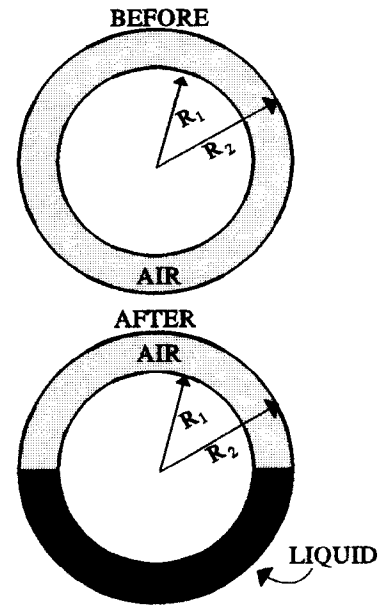


SENIOR PROJECT PHYSICS - TEST - CHAPTERS 20-21 - 1994-95

ELECTROSTATIC POTENTIAL AND CAPACITANCE - 100 PTS

CAREFULLY ANSWER FOUR [4] OF THE FOLLOWING PROBLEMS. BE SURE TO SHOW ALL WORK CAREFULLY! AN EXTRA PROBLEM MAY BE DONE FOR 5 POINTS EXTRA CREDIT!

1. Consider a capacitor consisting of two concentric cylindrical surfaces with radii of $R_1 = 5.00$ cm and $R_2 = 9.00$ cm as shown at the right. The concentric cylinders have a total length of $h = 225$ cm. The space between and around the surfaces is initially filled with air. A power supply, which has an EMF of 64.0 VDC, is connected between the two surfaces until the capacitor is fully charged.

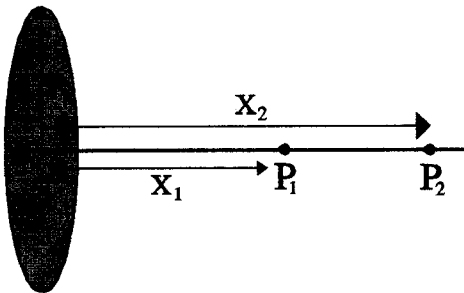


- What is the capacitance of this cylindrical capacitor? [5 pts] [Memory does not count! You MUST show where your answer comes from.]
- How much charge will be stored on each cylindrical shell after this capacitor is fully charged? [5 pts]
- How much energy will be stored in this capacitor when it is fully charged? [5 pts]

Suppose that half [1/2] of the space between the two shells is filled with an insulating liquid that has a dielectric constant of $K = 6.2$

- What will be the new capacitance of this cylindrical capacitor? [5 pts]
- How much charge will flow onto or off the capacitor as a result of the addition of this insulating liquid? [5 pts]

2. Consider a charged disk, which has a radius of $R = 8.0$ cm., and a charge density $\sigma = (3.0 + 220r) \mu\text{C}/\text{m}^2$, which varies with distance from the center of the disc.



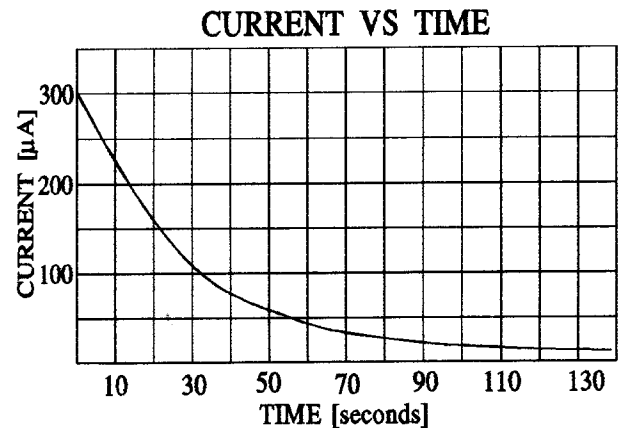
- What will be the electrostatic potential at point P_1 located at $X_1 = 12.0$ cm from the disk as shown in the diagram? [10 pts]
- How much work would be done in moving a $5.0 \mu\text{C}$ charge from infinity to point P_1 ? [5 pts]

Suppose that this $5.0 \mu\text{C}$ charge, which has a mass of 5.2×10^{-5} kg, at point P_1 is released and is allowed to accelerate to point P_2 , which is located a distance of $X_2 = 18.0$ cm from the disk.

- What will be the velocity of the charged particle when it reaches point P_2 ? [10 pts]

3. A capacitor C is connected to a DC power supply until the capacitor is completely charged. This capacitor is then removed from the power supply and is connected to a load resistance of $R = 13,600 \Omega$ through a sensitive galvanometer. The current flowing through the galvanometer, as a function of time, is displayed on the graph below.

- What is the time constant for this circuit? [5 pts]
- What is the value of the capacitor C ? [4 pts]
- What is the EMF of the DC power supply? [4 pts]
- How much charge will remain if this capacitor after 100 seconds? [4 pts]
- How much energy will be dissipated in this circuit during the first 100.0 seconds that this capacitor is discharging? [4 pts]
- What will be the current flowing in this circuit 100.0 seconds after discharge has begun? [4 pts]



4. Consider a capacitor C_1 which consists of two parallel plates, each with an area $A_1 = 4.1 \text{ m}^2$, separated by a distance $d_1 = .0015$ millimeters of air. This capacitor is connected to a DC power supply, which is generating a potential difference $V = 24.0$ Volts, until this capacitor is fully charged.

a. What is the capacitance of this capacitor? [5 pts]

This capacitor is then removed from the power supply and is then attached to a second, uncharged capacitor C_2 which consists of two parallel plates, each with an area $A_2 = 8.2 \text{ m}^2$, separated by a distance of $d_2 = .00050$ millimeters of air.

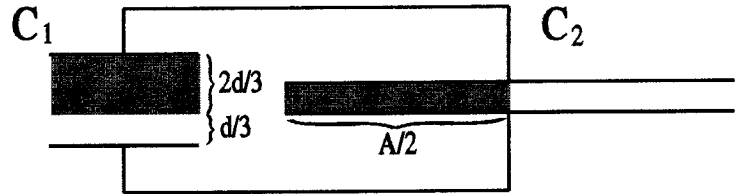
b. What will be the charge on C_1 AFTER it is connected to capacitor C_2 ? [5 pts]

c. How much energy is lost to this system in the process of charging capacitor C_2 ? [5 pts]

Suppose that an insulator with a dielectric constant $K = 5.2$ is inserted between the plates of capacitor C_1 so as to completely fill $2/3$ of the space between the plates as shown, while the same insulator is used to fill $1/2$ of the space between the plates of capacitor C_2 as shown.

d. What will be the total capacitance of this system after the addition of the insulating material? [5 pts]

e. What will be the final potential across these two capacitors? [5 pts]



5. Three charges are arranged as shown to the right where;

$$Q_1 = -48 \mu\text{C}, Q_2 = +32 \mu\text{C}, Q_3 = +16 \mu\text{C} \text{ and}$$

$$X_1 = 3.0 \text{ cm}, X_2 = 6.0 \text{ cm}, X_3 = 4.0 \text{ cm}.$$

a. What will be the electrostatic potential at point P? [5 pts]

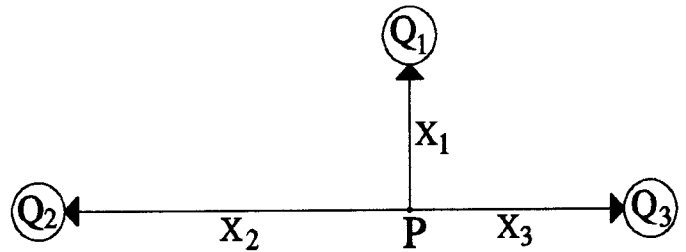
b. How much work would be required to assemble this system of particles? [5 pts]

Suppose that Q_1 could be moved onto a line connecting Q_2 and Q_3 .

c. Where could Q_1 be placed on this line such that its electrostatic potential is zero? [5 pts]

d. Where could Q_1 be placed on this line so as to feel zero electrostatic force? [5 pts]

e. How could these three charges be arranged such that the potential energy of this system of charges is zero [excluding infinity!]? [5 pts]



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1. $R_1 := 5.0 \cdot \text{cm}$ $R_2 := 9.0 \cdot \text{cm}$ $h := 225 \cdot \text{cm}$ $\text{EMF} := 64.0 \cdot \text{volt}$ $\epsilon_0 := 8.85 \cdot 10^{-12} \cdot \frac{\text{coul}^2}{\text{newton} \cdot \text{m}^2}$

$V := \text{EMF}$ $N := \text{newton}$ $C := \text{coul}$ $k := 9 \cdot 10^9 \cdot \frac{\text{m}^2}{\text{C}^2}$ $\mu := (1 \cdot 10)^{-6}$

$C = \frac{Q}{V}$ $\int E \, dA = 4 \cdot \pi \cdot k \cdot \lambda \cdot h$ $E = 4 \cdot \pi \cdot k \cdot \lambda \cdot \frac{h}{A}$ $A = 2 \cdot \pi \cdot r \cdot h$ $E = 4 \cdot \pi \cdot k \cdot \lambda \cdot \frac{h}{2 \cdot \pi \cdot r \cdot h}$ $E = 2 \cdot k \cdot \frac{\lambda}{r}$

$E = 2 \cdot k \cdot \frac{\lambda}{r}$ $\lambda = \frac{Q \text{ enclosed}}{h}$ $E = 2 \cdot k \cdot \frac{Q \text{ enclosed}}{(h \cdot r)}$

$V = \int_{R_2}^{R_1} 2 \cdot k \cdot \frac{Q \text{ enclosed}}{(h \cdot r)} \, dr$ $V = \left(2 \cdot k \cdot \frac{Q \text{ enclosed}}{h} \cdot \ln(R_1) - 2 \cdot k \cdot \frac{Q \text{ enclosed}}{h} \cdot \ln(R_2) \right)$

$V = \frac{2 \cdot k \cdot Q \text{ enclosed}}{h} \cdot \ln\left(\frac{R_2}{R_1}\right)$ $C = \frac{Q \text{ enclosed}}{V}$

$C = \frac{Q \text{ enclosed}}{\frac{2 \cdot k \cdot Q \text{ enclosed}}{h} \cdot \ln\left(\frac{R_2}{R_1}\right)}$ $C := \frac{h}{2 \cdot k \cdot \ln\left(\frac{R_2}{R_1}\right)}$ $C = 2.127 \cdot 10^{-10} \cdot \text{farad}$ **1a. The capacitance of the cylindrical capacitor. [5 pts]**

b. $Q \text{ enclosed} := C \cdot V$ $Q \text{ enclosed} = 1.361 \cdot 10^{-8} \cdot \text{coul}$ **1b. The total charge stored. [5 pts]**

c. $U := \frac{1}{2} \cdot C \cdot V^2$ $U = 4.355 \cdot 10^{-7} \cdot \text{joule}$ **1c. The energy stored in this capacitor. [5 pts]**

d. $K := 6.2$ $C_{\text{air}} := \frac{C}{2}$ $C_{\text{air}} = 1.063 \cdot 10^{-10} \cdot \text{farad}$ $C_{\text{liquid}} := \frac{C}{2} \cdot K$ $C_{\text{liquid}} = 6.593 \cdot 10^{-10} \cdot \text{farad}$

$C_{\text{new}} := C_{\text{air}} + C_{\text{liquid}}$ $C_{\text{new}} = 7.656 \cdot 10^{-10} \cdot \text{farad}$ **1d. The capacitance after adding the liquid dielectric. [5 pts]**

e. $Q_{\text{new}} := C_{\text{new}} \cdot V$ $Q_{\text{new}} = 4.9 \cdot 10^{-8} \cdot \text{coul}$

$\Delta Q := Q \text{ enclosed} - Q_{\text{new}}$ $\Delta Q = -3.539 \cdot 10^{-8} \cdot \text{coul}$ **1e. The charge that will flow onto the capacitor from the battery. [5 pts]**

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2. $\mu C := 1.0 \cdot 10^{-6} \cdot \text{coul}$ $r := 1$ $q := 5.0 \cdot \mu C$ $\sigma := 3.0 + 220 \cdot r$ $dV = \frac{k \cdot dq}{r}$ $dq = 2 \cdot \pi \cdot r \cdot dr$ $x := .12$
 $k := 9 \cdot 10^9$

a.

$$V := \int_0^{.08} \frac{(3.0 + 220 \cdot r) \cdot 2 \cdot 10^{-6} \cdot \pi \cdot k \cdot r}{\sqrt{r^2 + x^2}} dr \quad V = 1.988 \cdot 10^4 \quad V_1 := \underline{1.988 \cdot 10^4 \cdot \text{volt}} \quad \text{2a. The electrostatic potential at point } P_1. \text{ [10 pts]}$$

b.

$W_1 := V_1 \cdot q$ $W_1 = \underline{0.099 \cdot \text{joule}}$ **2b. Work that would be done in bringing a charge q from infinity to point P₁. [5 pts]**

c.

$x := .18$ $V_2 := \int_0^{.08} \frac{(3.0 + 220 \cdot r) \cdot 2 \cdot 10^{-6} \cdot \pi \cdot k \cdot r}{\sqrt{r^2 + x^2}} dr$ $V_2 = 1.404 \cdot 10^4$ $V_2 := \underline{1.404 \cdot 10^4 \cdot \text{volt}}$

$\Delta V := V_2 - V_1$ $\Delta V = -5.84 \cdot 10^3 \cdot \text{volt}$ $W := \Delta V \cdot q$ $W = -0.029 \cdot \text{joule}$

$m_c := 5.2 \cdot 10^{-5} \cdot \text{kg}$ $v := \sqrt{\frac{2 \cdot |W|}{m_c}}$ $v = \underline{33.512 \cdot \frac{\text{m}}{\text{sec}}}$ **2c. The velocity of the charged particle when it reaches point P₂. [10 pts]**

$$\int \frac{(k_1 + k_2 \cdot r) \cdot 2 \cdot \pi \cdot k \cdot r}{\sqrt{r^2 + x^2}} dr = \pi \cdot k \cdot k_2 \cdot r \cdot \sqrt{r^2 + x^2} - \pi \cdot k \cdot k_2 \cdot x^2 \cdot \ln\left(r + \sqrt{r^2 + x^2}\right) + 2 \cdot \pi \cdot k \cdot \sqrt{r^2 + x^2} \cdot k_1$$

$$691.15038378975451247 \cdot k \cdot r \cdot \sqrt{r^2 + x^2} - 691.15038378975451247 \cdot k \cdot x^2 \cdot \ln\left(r + \sqrt{r^2 + x^2}\right) + 18.849555921538759431 \cdot k \cdot \sqrt{r^2 + x^2}$$

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3. $\Omega := \text{ohm}$ $\mu\text{A} := 1 \cdot 10^{-6} \cdot \text{amp}$ $R := 13600 \cdot \Omega$ $I_0 := 300 \cdot \mu\text{A}$ $\mu\text{F} := 1 \cdot 10^{-6} \cdot \text{farad}$ $t := 1000 \cdot \text{sec}$

a. $I_{tc} := \frac{I_0}{e}$ $I_{tc} = 110.364 \cdot \mu\text{A}$ $t_c := \underline{30 \cdot \text{sec}}$ **3a. The time constant for this circuit. [5 pts]**

b. $t_c = R \cdot C$ $C := \frac{t_c}{R}$ $C = 0.002 \cdot \text{farad}$ $C = \underline{2.206 \cdot 10^3 \cdot \mu\text{F}}$ **3b. The capacitance of this capacitor. [4 pts]**

c. $\text{EMF} := I_0 \cdot R$ $\text{EMF} = \underline{4.08 \cdot \text{volt}}$ **3c. The EMF of the power supply. [4 pts]**

d.
$$\int I_0 \cdot e^{-\frac{t}{t_c}} dt = \left(-t_c \cdot \exp\left(\frac{-t}{t_c}\right) \cdot I_0 \right) \quad \left(-t_c \cdot \exp\left(\frac{-t}{t_c}\right) \cdot I_0 \right)$$

$t := \infty \cdot \text{sec}$ $Q_{\text{inf}} := \left(\left(-t_c \cdot \exp\left(\frac{-t}{t_c}\right) \cdot I_0 \right) \right)$ $Q_{\text{inf}} = 0 \cdot \text{coul}$

$t := 100 \cdot \text{sec}$ $Q_{100} := \left(-t_c \cdot \exp\left(\frac{-t}{t_c}\right) \cdot I_0 \right)$ $Q_{100} = 3.211 \cdot 10^{-4} \cdot \text{coul}$

$Q_{\text{remaining}} := Q_{100} - Q_{\text{inf}}$ $Q_{\text{remaining}} = \underline{321.066 \cdot \mu\text{C}}$ **3d. The charge remaining in the capacitor after discharging for t seconds. [4 pts]**

e. $Q_{\text{ini}} := I_0 \cdot t_c$ $Q_{\text{ini}} = 9 \cdot 10^3 \cdot \mu\text{C}$ $V_{\text{ini}} := \text{EMF}$ $V_{\text{final}} := \frac{Q_{100}}{C}$ $V_{\text{final}} = 0.146 \cdot \text{volt}$

$U_{\text{ini}} := \frac{1}{2} \cdot C \cdot V_{\text{ini}}^2$ $U_{\text{ini}} = 0.018 \cdot \text{joule}$ $U_{\text{final}} := \frac{1}{2} \cdot C \cdot V_{\text{final}}^2$ $U_{\text{final}} = 2.337 \cdot 10^{-5} \cdot \text{joule}$

$\Delta U := U_{\text{final}} - U_{\text{ini}}$ $\Delta U = \underline{-0.01834 \cdot \text{joule}}$ **3e. Energy lost in the process of discharging the capacitor for t seconds. [4 pts]**

f. $t := 100 \cdot \text{sec}$ $I_{100} := I_0 \cdot e^{-\frac{t}{t_c}}$ $I_{100} = \underline{10.702 \cdot \mu\text{A}}$ **3f. The current after 100 seconds. [4 pts]**

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4. $A_1 := 4.1 \cdot \text{m}^2$ $d_1 := 0.0015 \cdot \text{mm}$ $A_2 := 8.2 \cdot \text{m}^2$ $d_2 := 0.00050 \cdot \text{mm}$ $V := 24.0 \cdot \text{volt}$ $\epsilon_0 = 8.85 \cdot 10^{-12} \cdot \frac{\text{coul}^2}{\text{newton} \cdot \text{m}^2}$

a. $C_1 := \frac{\epsilon_0 \cdot A_1}{d_1}$ $C_1 = \underline{24.19 \cdot \mu\text{F}}$ **4a. The capacitance of the initial capacitor. [5 pts]**

b. $C_2 := \frac{\epsilon_0 \cdot A_2}{d_2}$ $C_2 = 145.14 \cdot \mu\text{F}$ $Q_2 := 1 \cdot \mu\text{C}$

$Q_1 := C_1 \cdot V$ $Q_1 = 580.56 \cdot \mu\text{C}$ $Q_t := Q_1$ $Q_t = 580.56 \cdot \mu\text{C}$

$V_1 = V_2$ $C = \frac{Q}{V}$ therefore $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ and $Q_1 := Q_t - Q_2$

GIVEN $\frac{Q_t - Q_2}{C_1} = \frac{Q_2}{C_2}$ $Q_2 := \text{FIND}(Q_2)$ $Q_2 = 497.623 \cdot \mu\text{C}$

$Q_1 := Q_t - Q_2$ $Q_1 = \underline{82.937 \cdot \mu\text{C}}$ **4b. The final charge stored on Q1. [5 pts]**

c. $U_{\text{ini}} := \frac{1}{2} \cdot C_1 \cdot V^2$ $U_{\text{ini}} = 0.00697 \cdot \text{joule}$

$V_1 := \frac{Q_1}{C_1}$ $V_1 = 3.429 \cdot \text{volt}$ $V_2 := \frac{Q_2}{C_2}$ $V_2 = 3.429 \cdot \text{volt}$

$U_{\text{final}} := \frac{1}{2} \cdot C_1 \cdot V_1^2 + \frac{1}{2} \cdot C_2 \cdot V_2^2$ $U_{\text{final}} = \underline{9.952 \cdot 10^{-4}} \cdot \text{joule}$

$\Delta U := U_{\text{final}} - U_{\text{ini}}$ $\Delta U = \underline{-0.00597} \cdot \text{joule}$ **4c. The energy lost in this process. [5 pts]**

4. [cont]

d. $K := 5.2$ $C_{1f} := 1 \cdot \mu\text{F}$ $C_{2f} := 1 \cdot \mu\text{F}$

$C_{11} := \frac{\epsilon_0 \cdot K \cdot A_1}{\left(\frac{2 \cdot d_1}{3}\right)}$ $C_{11} = 188.682 \cdot \mu\text{F}$ $C_{12} := \frac{\epsilon_0 \cdot A_1}{\left(\frac{d_1}{3}\right)}$ $C_{12} = 72.57 \cdot \mu\text{F}$

GIVEN $\frac{1}{C_{1f}} = \frac{1}{C_{11}} + \frac{1}{C_{12}}$ $C_{1f} := \text{FIND}(C_{1f})$ $C_{1f} = 52.412 \cdot \mu\text{F}$

$C_{21} := \frac{\epsilon_0 \cdot K \cdot \frac{A_2}{2}}{d_2}$ $C_{21} = 377.364 \cdot \mu\text{F}$ $C_{22} := \frac{\epsilon_0 \cdot \frac{A_2}{2}}{d_2}$ $C_{22} = 72.57 \cdot \mu\text{F}$

$C_{2f} := C_{21} + C_{22}$ $C_{2f} = 449.934 \cdot \mu\text{F}$

$C_{\text{tf}} := C_{1f} + C_{2f}$ $C_{\text{tf}} = \underline{502.346 \cdot \mu\text{F}}$ **4d. The final capacitance after adding the insulator. [5 pts]**

e. $V_f := \frac{Q_t}{C_{\text{tf}}}$ $V_f = \underline{1.156 \cdot \text{volt}}$ **4e. The final voltage across the capacitors. [5 pts]**

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5. $Q_1 := -48 \mu\text{C}$ $Q_2 := 32 \mu\text{C}$ $Q_3 := 16 \mu\text{C}$ $x_1 := 3.0 \text{ cm}$ $x_2 := 6.0 \text{ cm}$ $x_3 := 4.0 \text{ cm}$

$$V_{p1} := \frac{k \cdot Q_1}{x_1} \quad V_{p1} = -1.44 \cdot 10^7 \cdot \text{kg}^{-1} \cdot \text{m}^{-3} \cdot \text{vol} \quad V_{p2} := \frac{k \cdot Q_2}{x_2} \quad V_{p2} = 4.8 \cdot 10^6 \cdot \text{kg}^{-1} \cdot \text{m}^{-3} \cdot \text{vol} \quad V_{p3} := \frac{k \cdot Q_3}{x_3} \quad V_{p3} = 2.4 \cdot 10^6 \cdot \text{kg}^{-1} \cdot \text{m}^{-3} \cdot \text{vol}$$

a. $V_p := V_{p1} + V_{p2} + V_{p3}$ $V_p = -6 \cdot 10^6 \cdot \text{kg}^{-1} \cdot \text{m}^{-3} \cdot \text{vol}$ **5a. The potential at point p. [5 pts]**

b. $W_{12} := \frac{k \cdot Q_1 \cdot Q_2}{\sqrt{x_1^2 + x_2^2}}$ $W_{12} = -206.076 \cdot \text{kg}^{-1} \cdot \text{m}^{-3} \cdot \text{vol}^2 \cdot \frac{k \cdot Q_2 \cdot Q_3}{(x_2 + x_3)}$ $W_{23} = 46.08 \cdot \text{kg}^{-1} \cdot \text{m}^{-3} \cdot \text{sec}^2 \cdot \text{coul}^2 \cdot \text{joule}$

$$W_{13} := \frac{k \cdot Q_1 \cdot Q_3}{\sqrt{x_1^2 + x_3^2}} \quad W_{13} = -138.24 \cdot \text{kg}^{-1} \cdot \text{m}^{-3} \cdot \text{sec}^2 \cdot \text{coul}^2 \cdot \text{joule} \quad W_{13} = 3.6 \cdot 10^6 \cdot \text{kg}^{-1} \cdot \text{m}^{-3} \cdot \text{sec}^2 \cdot \text{coul}^2 \cdot \text{joule}$$

$W_{\text{total}} := W_{12} + W_{23} + W_{13}$ $W_{\text{total}} = -298.236 \cdot \text{kg}^{-1} \cdot \text{m}^{-3} \cdot \text{sec}^2 \cdot \text{coul}^2 \cdot \text{joule}$ **5b. Energy released in the process of assembling these three charges.**

c. $r := .03 \cdot \text{m}$ GIVEN $\frac{k \cdot Q_2}{r} = \frac{k \cdot Q_3}{(x_2 + x_3 - r)}$ $r := \text{FIND}(r)$ $r = 0.067 \cdot \text{m}$ from Q_3

$x_2 + x_3 - r = 0.033 \cdot \text{m}$ from Q_2 **5c. Location where Q1 could be placed so as to have zero potential. [5 pts]**

5. [cont]

d. GIVEN $\frac{k \cdot Q_2}{r^2} = \frac{k \cdot Q_3}{(x_2 + x_3 - r)^2}$ $r := \text{FIND}(r)$

$r = 0.059 \cdot \text{m}$ from Q_3 **5d. Place along the line joining Q_2 to Q_3 where**

$x_2 + x_3 - r = 0.041 \cdot \text{m}$ from Q_2 **Q_1 would feel no force. [5 pts]**

e. **Many different answers - student must demonstrate! [5 pts]**