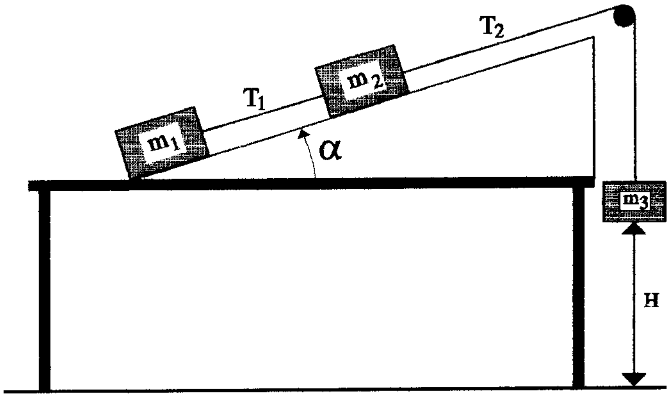
SCIENCE/ENGINEERING PHYSICS TEST - CHAPTERS 4-5 - 100 PTS - 1993-1994B

NEWTON'S LAWS OF MOTION

OF THE FOLLOWING PROBLEMS YOU MUST DO FOUR [4]. BE SURE TO SHOW ALL WORK CLEARLY AND COMPLETELY FOR FULL CREDIT. A 5TH PROBLEM MAY BE DONE FOR EXTRA CREDIT [5 pts].

1. A mass of $m_1 = 3.5$ kg is sitting at the bottom of an inclined plane which is 2.6 meters long and which is sitting on top of a table as shown to the right. This mass is, in turn, attached with a light string to a second mass of $m_2 =$ 5.0 kg which initially is sitting 1.0 meters from the bottom of the incline. A second string is attached to the second mass, is strung over a pulley, and is finally attached to a third mass of $m_3 = 9.2$ kg which is suspended at a height of H = 1.10 meters above the floor. Assume, initially, that there is no friction and that the angle between the incline and the tabletop is 28.0°.

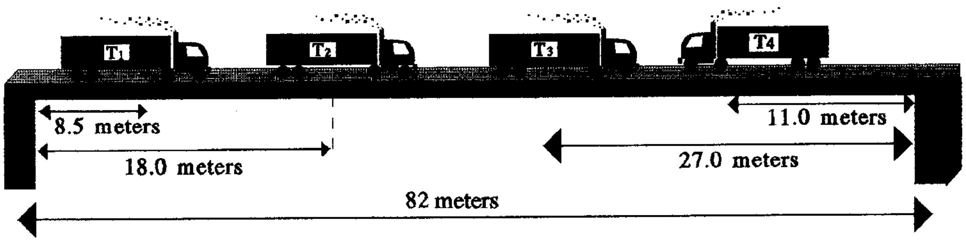


   a. Draw three free body diagrams indicating all of the forces acting on each of these masses. [6 pts - 2 pts each]
   b. What will be the resulting acceleration of this system? [4 pts]
   c. What will be the tension T_1 in the string connecting mass m_1 with mass m_2 ? [4 pts]

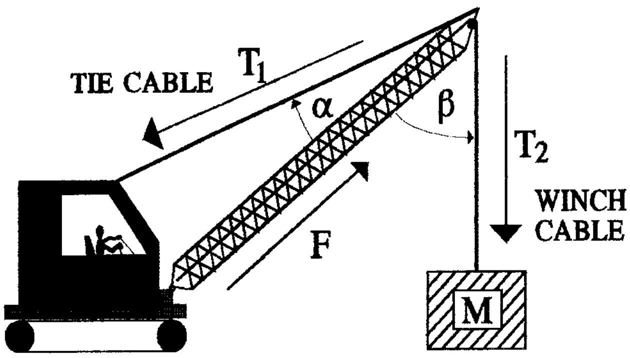
For the remainder of the problem assume that there is friction, that the system is initially at rest, that the coefficient of static friction is $\mu_s = .40$ and that the coefficient of kinetic friction is $\mu_k = .28$.

   d. What is the least amount of mass m_3 that can be added to this string to **START** this system moving? [4 pts]
   e. What will be the resulting acceleration of this system while being accelerated by this new mass m_3 ? [4 pts]
   f. What will be the tension T_2 in the string connecting mass m_2 to mass m_3 as this system accelerates? [3 pts]

2. A bridge consists of a long central span of 82 meters supported at each end by a pier. The bridge itself has a mass of 68,000 kg. On this bridge are four trucks. The first truck T_1 has a mass of 18,000 kg and is sitting 8.5 meters from the left end of the bridge, the second truck T_2 has a mass of 12,500 kg and is sitting 18.0 meters from the left end, the third truck T_3 has a mass of 21,000 kg and is sitting 27 meters from the right end of the bridge and finally the fourth truck T_4 has a mass of 15,000 kg and is sitting 11.0 meters from the right end of the bridge as shown below.
   a. Determine the upward forces, F_1 and F_2, exerted by each pier to support the bridge. [15 pts]
   b. Where along the length of this bridge could a single upward force be applied so as to lift the entire system in perfect equilibrium? [10 pts]

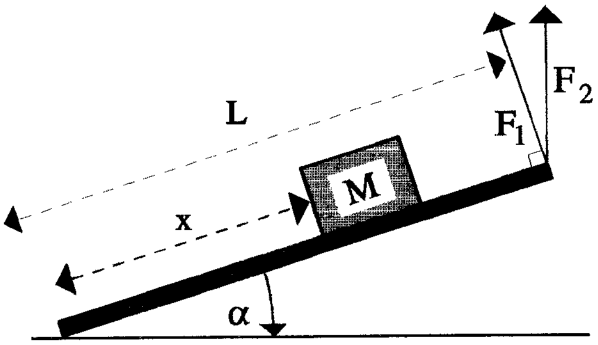


3. Consider a large crane, as shown below, which is being used to lift a heavy load of M = 18,500 kg. To the top of cabin of the crane there is attached a steel cable T_1 which is connected to the end of the boom. The angle between cable T_1 and the boom is $\alpha = 25°$. A second cable T_2 has one end attached to the load while the other end of the cable is attached to a winch at the base of the cabin after passing over a large pulley at the upper end of the boom. The angle between cable T_2 and the boom is $\beta = 48°$. The mass M is being lifted upward at a constant speed.



   a. What will be the tension T_2 in the cable lifting the load? [5 pts]
   b. What will be the tension T_1 in the tie cable? [10 pts]
   c. What will be the magnitude of the thrust force F being exerted by the boom? [10 pts]

4. A board, which has a mass of $m = 18.5$ kg and which is $L = 3.6$ meters long, is sitting on a horizontal surface. A crate, which has a mass of $M = 66.0$ kg, is sitting on the board a distance of $x = 2.5$ meters from the left end of the board. You grab the right end of the board and lift straight up until the board makes an angle of $\alpha = 24°$ with the floor. The coefficient of static friction between the crate and the board is $\mu_s = .53$ and the coefficient of kinetic friction is $\mu_k = .44$.

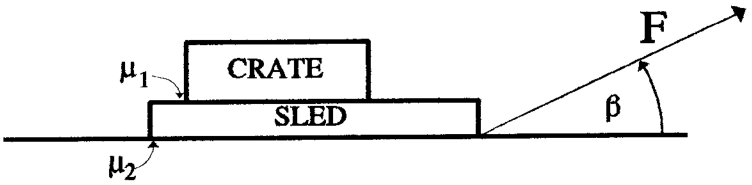


a. What will be the magnitude of the upward force F_2 needed to support this board? [5 pts]
b. What would be the magnitude of the force required to support this board if the force applied is exerted perpendicularly to the board as represented by F_1? [5 pts]

Suppose now that the end of the board is lifted until the crate begins to slide down the incline.

c. What will be th angle α between the board and the floor just as the crate begins to slide? [5 pts]
d. What will be the rate of acceleration of this mass as it slides down the incline? [5 pts]
e. What will be the velocity of the crate as it reaches the bottom of the incline? [5 pts]

5. A rope is being used to pull a sled along a horizontal surface as shown to the right. The sled has a mass of 18.0 kg. and sitting on the sled is a crate, which has a mass of 46.0 kg. The coefficient of friction between the sled and the crate is $\mu_1 = .34$ while the coefficient of friction between the sled and the ground is $\mu_2 = .62$. The angle between the rope and the horizontal is $\beta = 31°$ as shown.

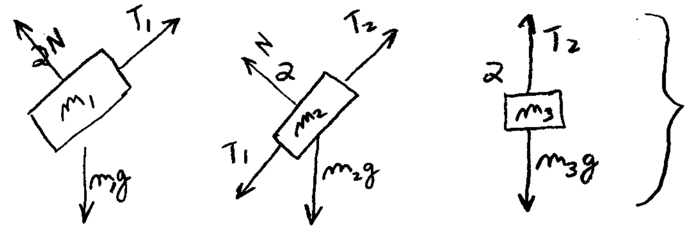


a. How much force F must be applied to this rope in order to pull the sled along at a constant speed? [5 pts]
b. With what maximum acceleration can this sled be pulled without the crate sliding off the sled? [5 pts]
c. What maximum force F can be applied to this sled without the crate slipping? [5 pts]

Suppose now that the applied force is increased to $F = 815$ N so as to accelerate the sled such that the crate slides relative to the sled.

d. Complete and label separate freebody diagrams for both the crate and the sled including all of the external forces acting on each. [5 pts]
e. What will be the acceleration of the sled if the applied force is $F = 815$ N? [5 pts]

1.



(a)

(b)

$$m_3 g - (m_1 + m_2) g \sin\alpha = (m_1 + m_2 + m_3) a$$

$$\underbrace{9.2(9.8)}_{90.16} - \underbrace{(8.5)9.8 \sin 28}_{39.11} = \underbrace{(9.2 + 3.5 + 5.0)}_{17.70} a \qquad \boxed{a = 2.88 \text{ m/s}^2}$$ (b) 4

(c) $T_1 - m_1 g \sin\alpha = m_1 a$

$$T_1 = m_1 a + m_1 g \sin\alpha = \underbrace{3.5(2.88)}_{10.08} + \underbrace{3.5(9.8)\sin 28}_{16.10} = 26.18 \text{ N} = \boxed{26.2 \text{ N}}$$ (c) 4

(d) $\Sigma F = 0$

$$m_3 g - (m_1 + m_2) g \sin\alpha - (m_1 + m_2) g \mu \cos\alpha = 0 \qquad m_3 = \frac{39.11 + 29.42}{9.8} = 6.993 = \boxed{7.0 \text{ kg}}$$ (d) 4

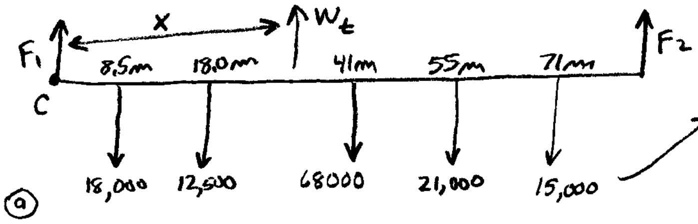
$$m_3(9.8) - \underbrace{8.5(9.8)\sin 28}_{39.11} - \underbrace{8.5(9.8)(.40)\cos 28}_{29.42} = 0$$

(e) $\Sigma F = ma$

$$\underbrace{7.0(9.8)}_{68.60} - \underbrace{8.5(9.8)\sin 28}_{39.11} - \underbrace{8.5(9.8)(.28)\cos 28}_{20.59} = \underbrace{(3.5 + 5.0 + 7.0)}_{15.5} a' \qquad a' = \boxed{.574 \text{ m/s}^2}$$ (e) 4

(e) $m_3 g - T_2 = m_3 a \qquad T_2 = m_3 g - m_3 a = m_3(g - a) = 7.0(9.8 - .574) = \boxed{64.6 \text{ N}}$ (f) 3

2.



TOTAL WEIGHT $= 1.318 \times 10^6$ N $= W_t$

(a)

$$5.386 \times 10^6 = 18{,}000(8.5) + 12{,}500(18) + 68{,}000(41) + 21{,}000(55) + 15{,}000(71) = F_2(82)$$

$$\frac{9.8(1.53 \times 10^5 + 2.25 \times 10^5 + 2.788 \times 10^6 + 1.155 \times 10^6 + 1.065 \times 10^6)}{82} = F_2 \Rightarrow \boxed{6.44 \times 10^5 \text{ N}}$$ (a)

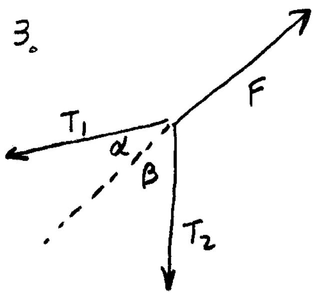
$$F_1 = W_t - F_2 = 1.318 \times 10^6 - 6.57 \times 10^5 \text{ N}$$

$$F_1 \Rightarrow \boxed{6.74 \times 10^5 \text{ N}}$$ 15

(b) $\Sigma \tau^+ = 5.281 \times 10^7$ Nm $= 1.318 \times 10^6 \, x$

$$x = 40.07 = \boxed{40.1 \text{ m FROM LEFT END}}$$ (b) 10

3.



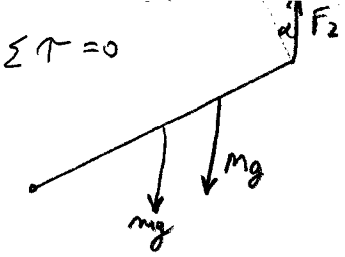
$T_2 \cos\beta + T_1 \cos\alpha = F$ (a) $T_2 = mg = 18{,}500(9.8) = \boxed{1.813 \times 10^5 \text{ N}}$ (a) 5

$T_2 \sin\beta = T_1 \sin\alpha$

(b) $T_1 = T_2 \dfrac{\sin\beta}{\sin\alpha} = 1.813 \times 10^5 \left(\dfrac{\sin 48}{\sin 25}\right) = \boxed{3.188 \times 10^5 \text{ N}}$ (b) 10

(c) $F = \underbrace{1.813 \times 10^5 \cos 48}_{1.213 \times 10^5} + \underbrace{3.188 \times 10^5 \cos 25}_{2.889 \times 10^5} = \boxed{4.10 \times 10^5 \text{ N}}$ (c) 10

4. $\Sigma \tau = 0$



$mg\frac{L}{2}\cancel{\cos\alpha} + Mg\,x\,\cancel{\cos\alpha} = F_2 L\cancel{\cos\alpha}$

(a) $F_2 = \dfrac{\frac{mgL}{2} + Mgx}{L} = \dfrac{mg}{2} + \dfrac{Mgx}{L} = \overbrace{\dfrac{18.5(9.8)}{2}}^{90.65} + \overbrace{\dfrac{66(9.8)(2.5)}{3.6}}^{449.2}$

$\boxed{F_2 = 540N}$ (a) 5

(b) SAME AS (a) EXCEPT cos α DOES NOT CANCEL!

$mg\frac{L}{2}\cos\alpha + Mgx\cos\alpha = F_1 L$

$F_1 = \dfrac{mg\cos\alpha}{2} + \dfrac{Mgx\cos\alpha}{L} = \overbrace{\dfrac{18.5(9.8)}{2}\cos 31}^{77.7} + \overbrace{\dfrac{66(9.8)(2.5)\cos 31}{3.6}}^{385}$

$\boxed{F_1 = 463N}$ (b) 5

(c) $\cancel{mg}\mu\cos\theta = \cancel{mg}\sin\beta$

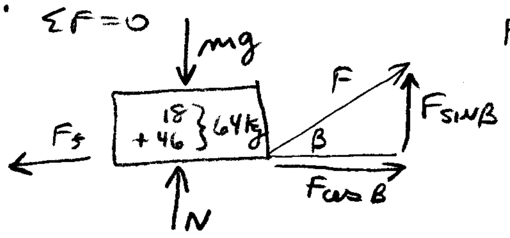
$\tan\alpha = \mu = .53$

$\boxed{\alpha = 27.9^\circ}$ (c) 5

(d) $\Sigma F = ma \Rightarrow \cancel{m}g\sin\alpha - \cancel{m}g\mu\cos\alpha = \cancel{m}a = \overbrace{9.8\sin 27.9}^{4.586} - \overbrace{9.8(.44)\cos 27.9}^{3.811} = .7750 = \boxed{.78 m/s}$ (d) 5

(e) $D = \frac{1}{2}at^2 = 2.5 = \frac{.78}{2}t^2$ $\quad t = 2.53 s$ $\quad v = at + v_0 = .78(2.53) = \boxed{1.975 m/s}$ (e) 5

5. $\Sigma F = 0$



$F\cos\beta = F_f = N\mu$ + $N + F\sin\beta = mg$

$N = mg - F\sin\beta$

(a)

$F\cos\beta = (mg - F\sin\beta)\mu$

$F\cos\beta = mg\mu - F\mu\sin\beta$

$F\cos\beta + F\mu\sin\beta = mg\mu$

$F(\cos\beta + \mu\sin\beta) = mg\mu$

$F = \dfrac{mg\mu}{(\cos\beta + \mu\sin\beta)} = \dfrac{64(9.8)(.62)}{(\cos 31 + .62\sin 31)} = \boxed{330.5 N}$ (a) 5

(b) $\Sigma F = 0$

$\cancel{m}g\mu = \cancel{m}a$

$a = g\mu = .34(9.8) = \boxed{3.33 m/s^2}$ (b) 5

(c) $\Sigma F = ma$ $\qquad F_f = (mg - F\sin\beta)\mu$

$F\cos\beta - F_f = ma$ ← TOTAL MASS

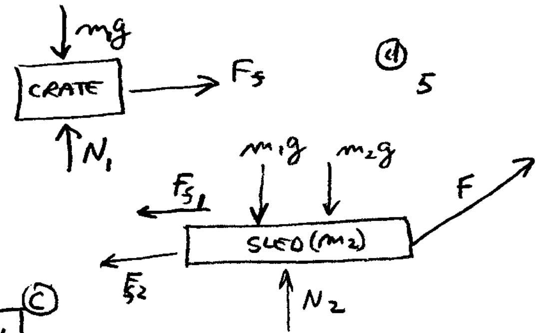
$F\cos\beta - (mg - F\sin\beta)\mu = ma$

$F\cos\beta - mg\mu + F\mu\sin\beta = ma$

$F(\cos\beta + \mu\sin\beta) = ma + mg\mu_2$

$F = \dfrac{mg\mu_2 + ma}{\cos\beta + \mu\sin\beta} = \dfrac{64(9.8)(.62) + 64(3.33)}{\cos 31 + .62(\sin 31)} = \boxed{512 N}$ (c) 5

(d)



(d) 5

(e) $\Sigma F = ma$

$F_{f_1} = N_1\mu_1 = m_1 g\mu_1$

$F\cos\beta - F_{f_2} - F_{f_1} = m_2 a$

$F_{f_2} = N_2\mu_2 = [(m_1+m_2)g - F\sin\beta]\mu$

$F\cos\beta - [(m_1+m_2)g - F\sin\beta]\mu_2 - m_1 g\mu_1 = m_2 a$

$\underbrace{815\cos 31}_{699 N} - \underbrace{[64(9.8) - 815\sin 31].62}_{129N} - \underbrace{46(9.8)(.34)}_{153 N} = 18a$

$\boxed{a = 23.2 m/s^2}$ (e) 5