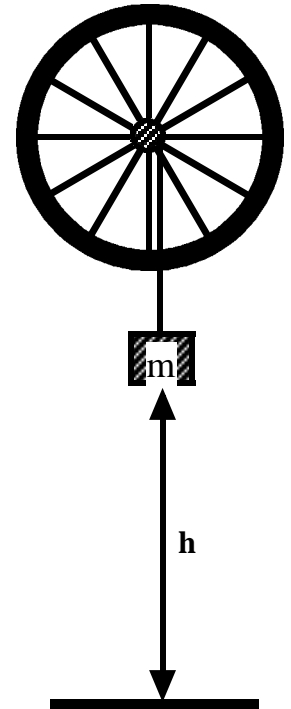


ROTATIONAL MOTION

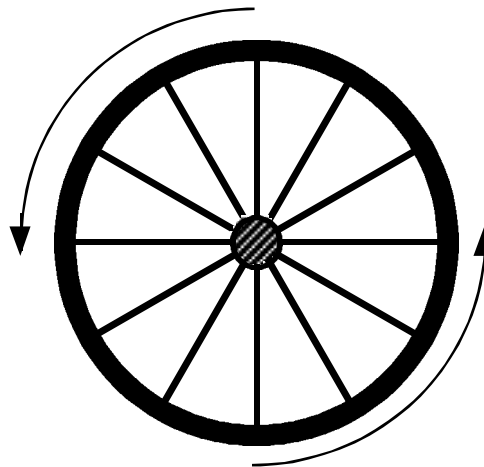
ANSWER EACH OF THE FOLLOWING PROBLEMS! BE SURE TO SHOW ALL WORK CAREFULLY!

1. A bicycle wheel is mounted as in the lab and as shown to the right. This wheel has a mass of 6.35 kg, a radius of $R = 35.0$ cm and is in the shape of a ring. A mass $m = 1.25$ kg is attached to the end of a string which is wrapped around an inner hub which has a radius $r = 1.70$ cm. Initially, the mass M is a distance $h = 77.0$ cm above the floor. [Assume friction is negligible!]
 - A. What is the moment of inertia of this wheel? [5 pts]
 - B. What is the magnitude of the torque applied to the wheel by the falling mass ? [5 pts]
 - C. What will be the resulting angular acceleration of this wheel? [5 pts]
 - D. What will be the angular velocity of this wheel 3.0 seconds after the falling mass is released ? [5 pts]
 - E. What will be the linear velocity of a point on the outer edge of the wheel 3.0 seconds after the falling mass has been released ? [5 pts]
 - F. What will be the velocity of the falling mass 3.0 seconds after being released ? [5 pts]
 - G. Through what angle will this wheel have rotated 3.0 seconds after the falling mass has been released ? [5 pts]
 - H. What will be the kinetic energy of this wheel 3.0 seconds after the mass has been released? [5 pts]
 - I. How much work was done on the wheel during the first 3.0 seconds? [5 pts]
 - J. How long will it take for the mass to reach the floor ? [5 pts]

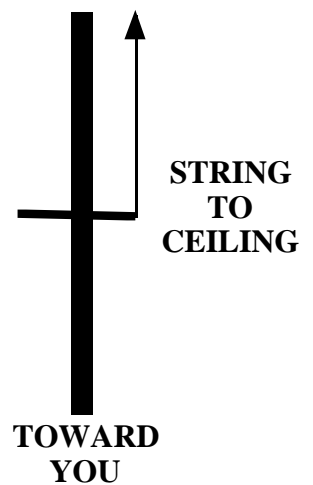


2. Consider a gyroscope which consists of a large wheel [ring] which has a mass of 8.50 kg and a radius of 28.0 cm. This wheel does 8.5 complete rotations each second and is rotating such that an observer from the left side sees the wheel rotating counter-clockwise as shown in the diagram to the right. When viewed from in front the top edge of the wheel is moving away from you.

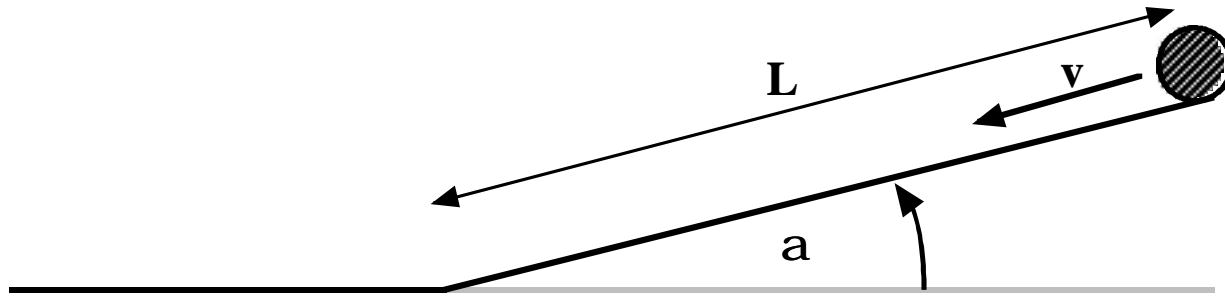
VIEW FROM LEFT SIDE



AWAY FROM YOU



- a. What is the angular velocity of this wheel? [5 pts]
- b. Using the diagram to the right determine the direction of the initial angular momentum of this wheel. Explain your answer clearly. [5 pts]
- c. What will be the magnitude of the angular momentum of this wheel? [5 pts]
- d. Suppose that a string is tied to the right end of the axle as shown so as to support the weight of the wheel. What will be the direction of the resulting angular precession? Explain carefully using a clear, well labeled diagram to support your answer! [5 pts]

ROTATIONAL MOTION

3. A sphere, which has mass of $m = 5.45$ kg and a radius of $r = 15.0$ cm, is sitting at the top of an inclined plane, which has a length of $L = 2.6$ meters and which meets the horizontal at an angle of $\alpha = 23.0^\circ$ as shown. This mass is released and is allowed to roll to the bottom of the incline without slipping.
- What is the moment of inertia of this sphere? [5 pts]
 - What will be the linear velocity of the sphere when it reaches the bottom of the incline? [5 pts]
 - What will be the angular velocity of the sphere when it reaches the bottom of the incline? [5 pts]
 - How long will it take for the sphere to reach the bottom of the incline? [5 pts]
 - How much work was done to the sphere as it rolled to the bottom of the incline? [5 pts]
 - What was the average acceleration of the sphere as it rolled to the bottom of the incline? [5 pts]
 - What average torque was applied to the sphere as it accelerated down the incline? [5 pts]
 - If this had been a disk instead of a sphere, would the linear velocity at the bottom been less than, more than or the same as for the sphere? Explain but do not use calculations! [5 pts]

1. $m_{\text{ring}} := 6.35 \cdot \text{kg}$ $r_{\text{ring}} := 35.0 \cdot \text{cm}$ $m_1 := 1.25 \cdot \text{kg}$ $r_{\text{hub}} := 1.70 \cdot \text{cm}$ $h := 77 \cdot \text{cm}$

a. $I_{\text{ring}} := m_{\text{ring}} \cdot r_{\text{ring}}^2$ $I_{\text{ring}} = 0.778 \cdot \text{kg} \cdot \text{m}^2$ **1a. Moment of inertia of the ring. [5 pts]**

b. $T := m_1 \cdot g \cdot r_{\text{hub}}$ $T = 0.208 \cdot \text{newton} \cdot \text{m}$ **1b. Torque applied to the wheel. [5 pts]**

c. $\alpha := \frac{T}{I_{\text{ring}}}$ $\alpha = 0.268 \cdot \frac{\text{rad}}{\text{sec}^2}$ **1c. The angular acceleration of the wheel. [5 pts]**

d. $t := 3.0 \cdot \text{sec}$ $\omega_f := \alpha \cdot t$ $\omega_f = 0.804 \cdot \frac{\text{rad}}{\text{sec}}$ **1d. Angular velocity of the wheel after t seconds. [5 pts]**

e. $v := \omega_f \cdot r_{\text{ring}}$ $v = 0.281 \cdot \frac{\text{m}}{\text{sec}}$ **1e. Linear velocity of a point on the outer edge of the wheel. [5 pts]**

f. $v_{m1} := \omega_f \cdot r_{\text{hub}}$ $v_{m1} = 0.014 \cdot \frac{\text{m}}{\text{sec}}$ **1f. Velocity of the falling mass. [5 pts]**

g. $\theta := \frac{1}{2} \cdot \alpha \cdot t^2$ $\theta = 1.206 \cdot \text{rad}$ **1g. Angular displacement of the wheel. [5 pts]**

h. $KE := \frac{1}{2} \cdot I_{\text{ring}} \cdot \omega_f^2$ $KE = 0.251 \cdot \text{joule}$ **1h. Kinetic energy of the wheel. [5 pts]**

i. $W := T \cdot \theta$ $W = 0.251 \cdot \text{joule}$ **1i. Work done on the wheel by the weight. [5 pts]**

j. GIVEN $h = \frac{1}{2} \cdot \alpha \cdot r_{\text{hub}} \cdot t^2$ $t := \text{FIND}(t)$ $t = 18.4 \cdot \text{sec}$ **1j. The initial height of the falling mass. [5 pts]**

2. $m_{\text{ring}} := 8.50 \cdot \text{kg}$ $r_{\text{ring}} := 28 \cdot \text{cm}$ $n := 8.5$

a. $\omega := \frac{2 \cdot \pi \cdot n}{1 \cdot \text{sec}}$ $\omega = 53.4 \cdot \frac{\text{rad}}{\text{sec}}$ **2a. The angular velocity of the wheel. [5 pts]**

b. The angular momentum of the wheel is left or out of the paper. [5 pts]

$$I := m_{\text{ring}} \cdot r_{\text{ring}}^2 \quad I = 0.666 \cdot \text{kg} \cdot \text{m}^2$$

c. $L := I \cdot \omega$ $L = 35.6 \cdot \text{kg} \cdot \frac{\text{m}^2}{\text{sec}}$ **2c. The angular momentum of the wheel. [5 pts]**

d. The wheel will rotate counterclockwise when viewed from above. [5 pts]

3. $m_{\text{sphere}} := 5.45 \cdot \text{kg}$ $r_{\text{sphere}} := 15.0 \cdot \text{cm}$ $L := 2.6 \cdot \text{m}$ $\beta := 23 \cdot \text{deg}$ $h := L \cdot \sin(\beta)$ $h = 1.016 \cdot \text{m}$ $v_{\text{sphere}} := 1 \cdot \frac{\text{m}}{\text{sec}}$

a. $I_{\text{sphere}} := \frac{2}{5} \cdot m_{\text{sphere}} \cdot r_{\text{sphere}}^2$ $I_{\text{sphere}} = 0.049 \cdot \text{kg} \cdot \text{m}^2$ **3a. The moment of inertia of the sphere. [5 pts]**

b. GIVEN $m_{\text{sphere}} \cdot g \cdot h = \frac{1}{2} \cdot m_{\text{sphere}} \cdot v_{\text{sphere}}^2 + \frac{1}{2} \cdot I_{\text{sphere}} \cdot \frac{v_{\text{sphere}}^2}{r_{\text{sphere}}^2}$

$v_{\text{sphere}} := \text{FIND}(v_{\text{sphere}})$ $v_{\text{sphere}} = 3.77 \cdot \frac{\text{m}}{\text{sec}}$ **3b. The velocity of the sphere when it reaches the bottom. [5 pts]**

c. $\omega_f := \frac{v_{\text{sphere}}}{r_{\text{sphere}}}$ $\omega_f = 25.15 \cdot \frac{\text{rad}}{\text{sec}}$ **3c. The final angular velocity of the sphere. [5 pts]**

d. $t := \frac{L}{\left(\frac{v_{\text{sphere}}}{2}\right)}$ $t = 1.378 \cdot \text{sec}$ **3d. The time for the sphere to roll to the bottom of the incline. [5 pts]**

e. $\text{Work} := m_{\text{sphere}} \cdot g \cdot h$ $\text{Work} = 54.3 \cdot \text{joule}$ **3e. The work done on the sphere as it rolls to the bottom. [5 pts]**

f. $a := \frac{v_{\text{sphere}} - 0 \cdot \frac{\text{m}}{\text{sec}}}{t}$ $a = 2.74 \cdot \frac{\text{m}}{\text{sec}^2}$ **3f. The linear acceleration of the sphere. [5 pts]**

g. $\alpha := \frac{a}{r_{\text{sphere}}}$ $T := I_{\text{sphere}} \cdot \alpha$ $T = 0.895 \cdot \text{N} \cdot \text{m}$ **3g. The net torque on the sphere. [5 pts]**

h. The linear velocity of a disk would have been less than a sphere because more energy is tied up in its rotational motion due to its higher moment of inertia. [5 pts]